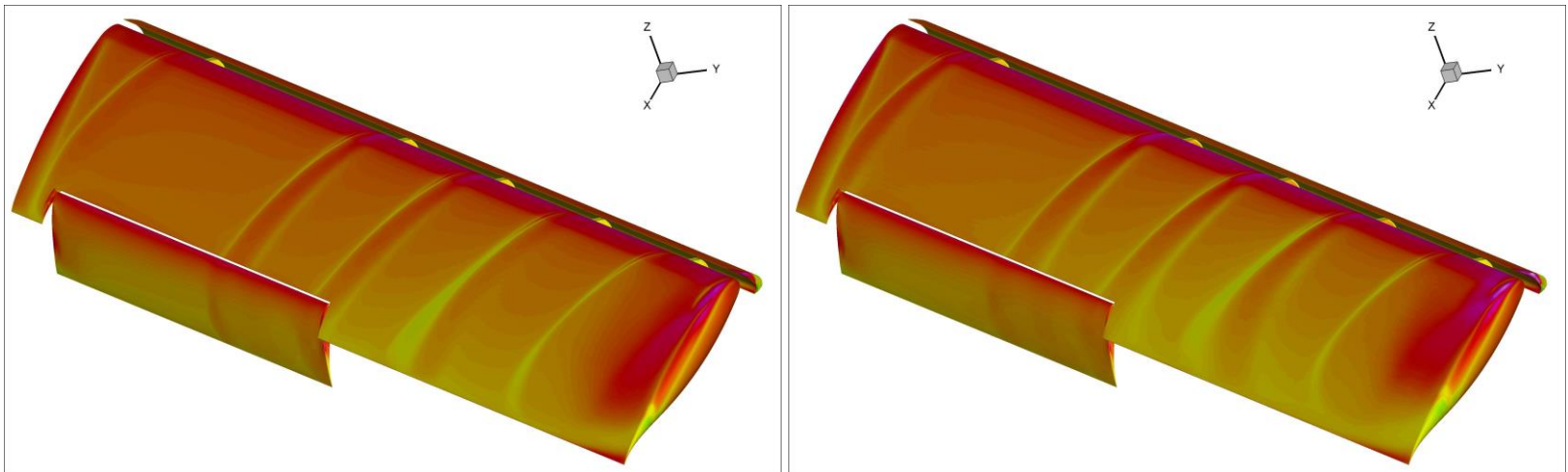




Application of HEMLAB Algorithm to HLPW6-TC1 with NASA Refine Anisotropic Mesh Refinement



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Numerical Method: HEMLAB Solver

Spatial Discretisation

- Vertex-centred (median-dual) finite-volume method
- Tet, prism, pyramid & hex elements – hybrid unstructured
- Edge-based data structure (cache-optimised, MPI-parallel)
- Inviscid fluxes: Roe, HLLC, AUSM⁺-up, AUSM⁺M, HLLC-LM
- 1st/2nd/3rd-order unweighted least-squares reconstruction
- Exact inviscid Jacobian evaluation with Tapenade library
- Limiters: Barth-Jespersen, Venkatakrishnan, Nishikawa
- Present results: 2nd-order Roe, no limiter

Nonlinear Solver

- Monolithic (fully coupled) approach
- Exact Jacobian (parallel block matrix) assembly
- Pseudo transient continuation (PTC)
- Newton method with line-search algorithm
- GMRES + Additive Schwarz / ILU(k) preconditioner
- METIS partitioning + RCM reordering
- PETSc SNES nonlinear solver

Viscous Fluxes & Turbulence

- Green–Gauss with edge-based gradients
- Avoids velocity–gradient decoupling
- Negative Spalart–Allmaras (SA-neg) model
- Exact wall-distance via Alternating Digital Tree (ADT)
- Analytical exact Jacobian



Numerical Method: Metric-Based Anisotropic Mesh Adaptation

NASA Refine (Langley)

- Riemannian metric tensor: $M(x) = R^T \wedge R$
- Edge split/collapse/swap + node smoothing
- EGADS/EGADSlite geometry kernel (CAD-consistent)
- Fully parallel (MPI) – capable of 100M+ element meshes
- Load balancing: METIS / Zoltan
- Feature based metric construction
- Parallel old to new mesh interpolation for mixed elements
- Tetrahedral elements with all solid nodes are avoided!

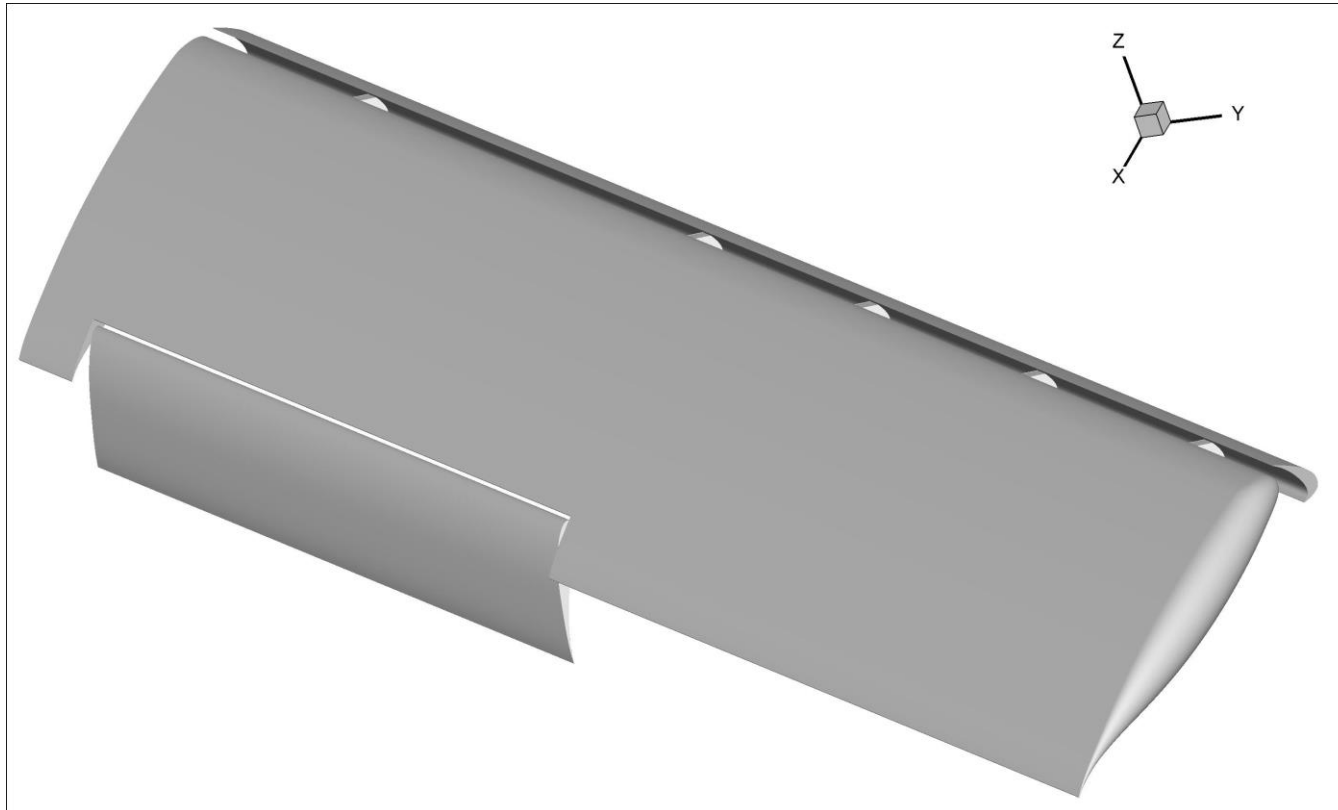
Sensor Functions

- Mach number Hessian or Mach + exponential wall-distance function
- L^p -norm error estimator ($p = 2$ or 4)
- Identifies shocks, wakes, boundary layer regions
- Entropy sensor is also very effective for capturing rotor wakes but not used in here



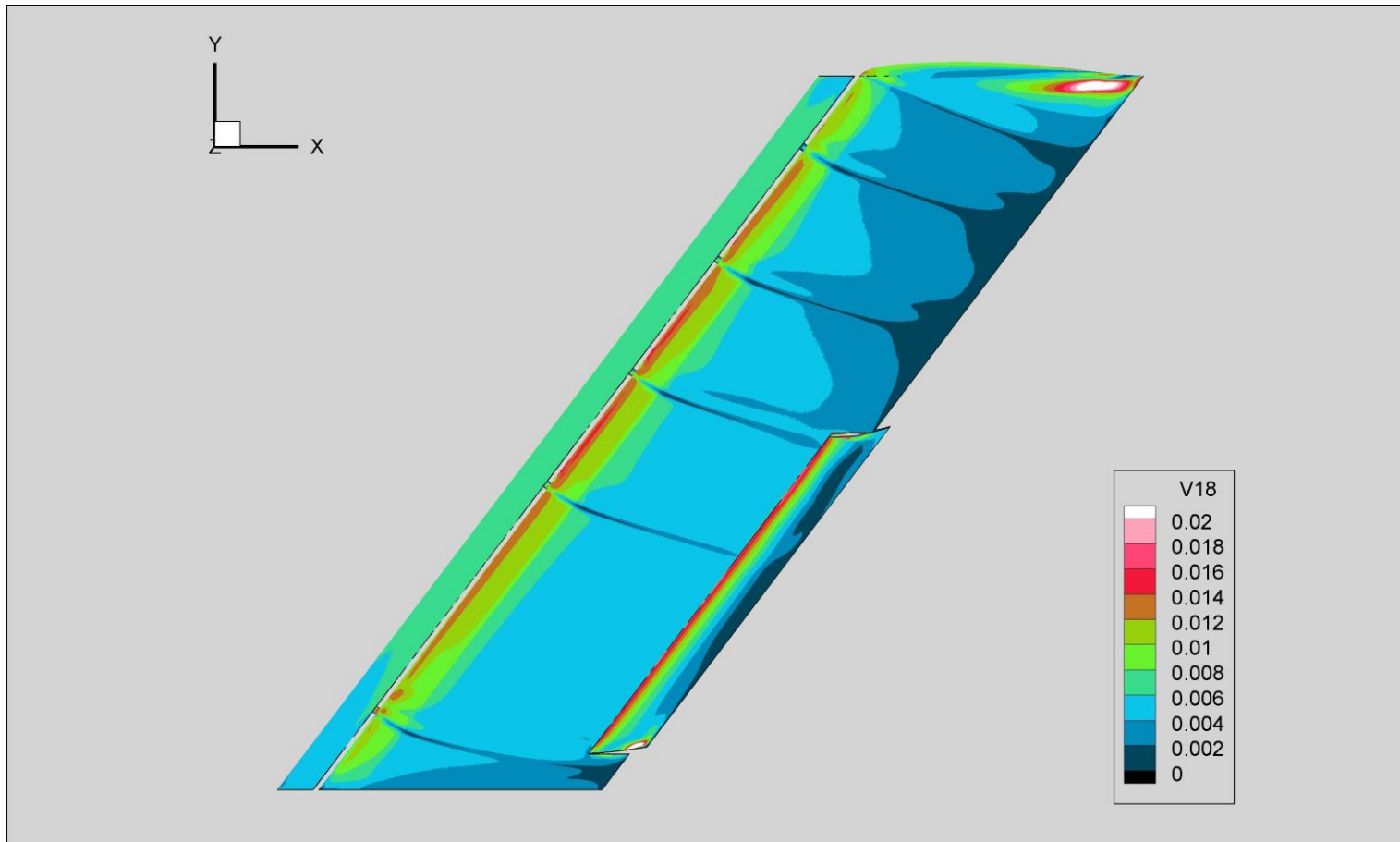
Test Case 1

Mach Number	0.20
Chord Reynolds Number	3.55×10^6
Angles of Attack	$10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ$
Reference Static Temperature	518.67°R
Reference Static Pressure	14.696 psi





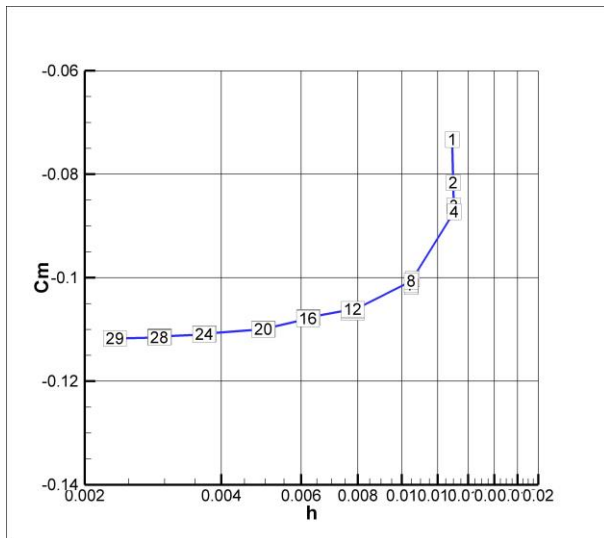
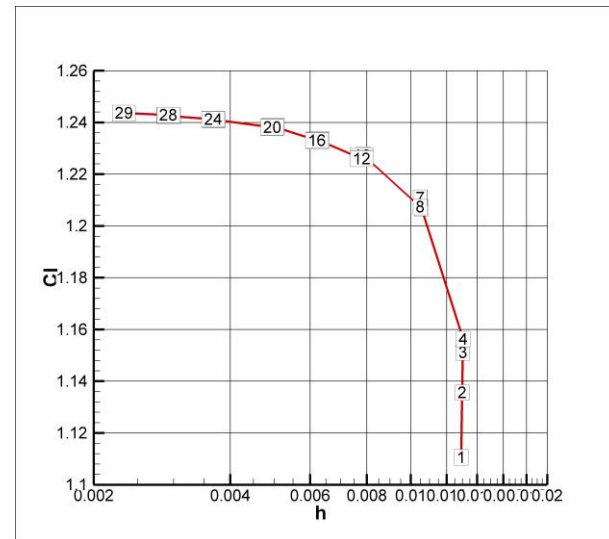
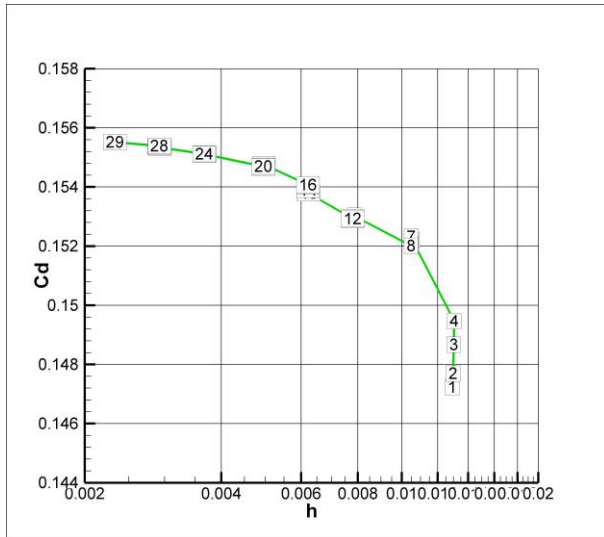
Surface Skin Friction Coefficient at Alpha=10 degrees



29th iteration (78M vertices)

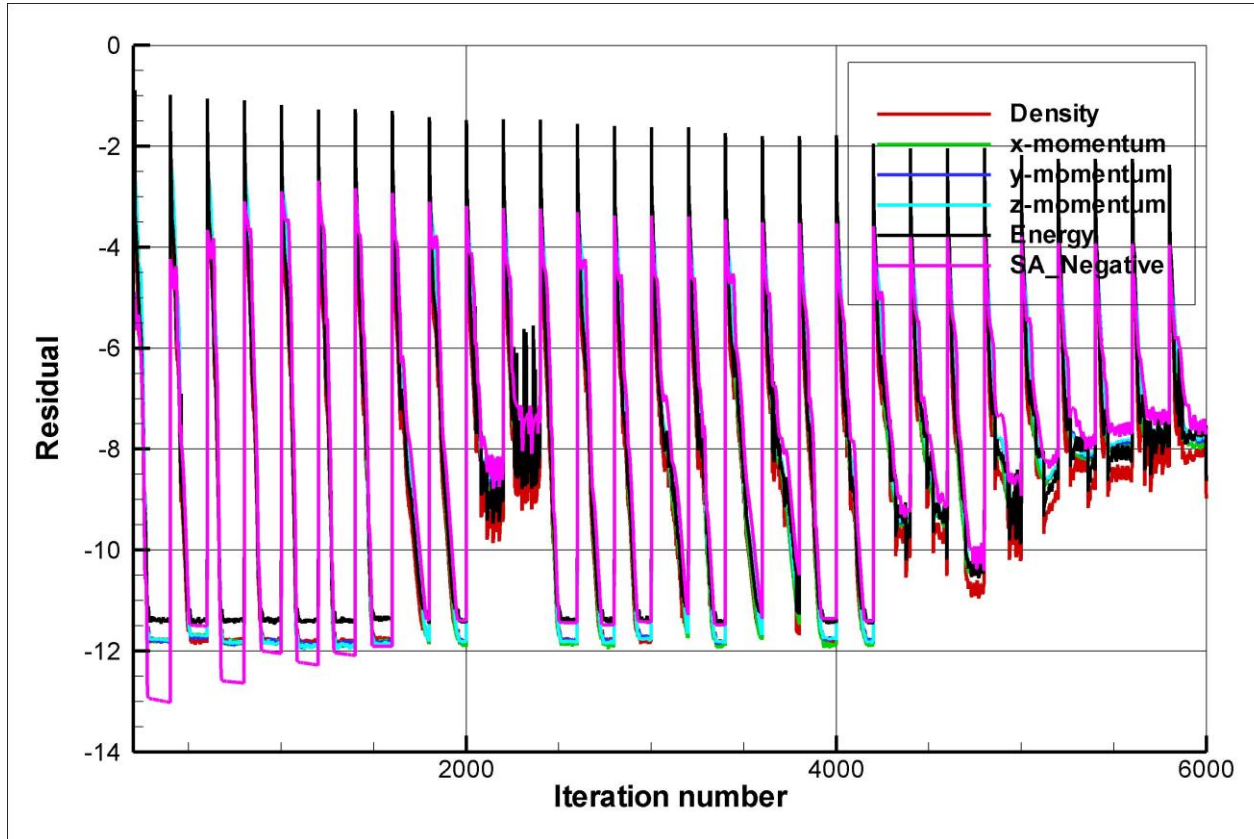


Convergence of Aerodynamics Loads with Mesh Refinement



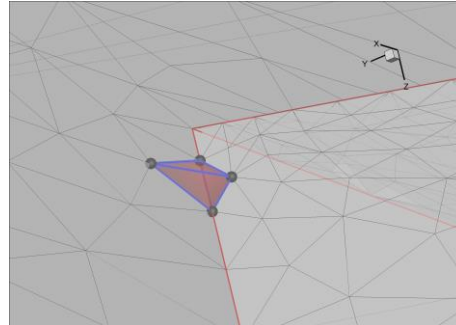
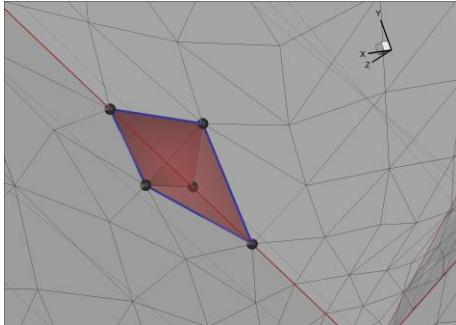


Residual Convergence with Mesh Refinement

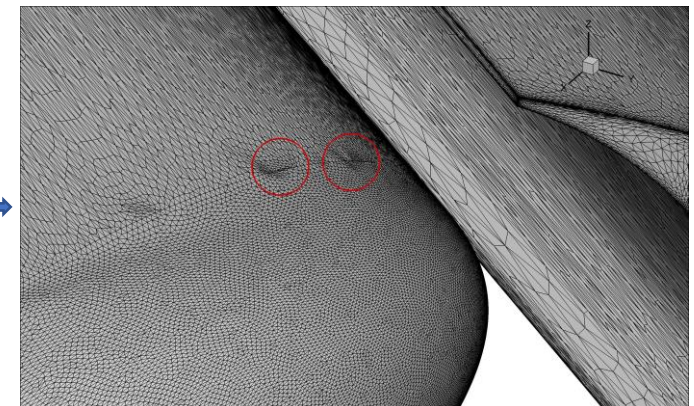
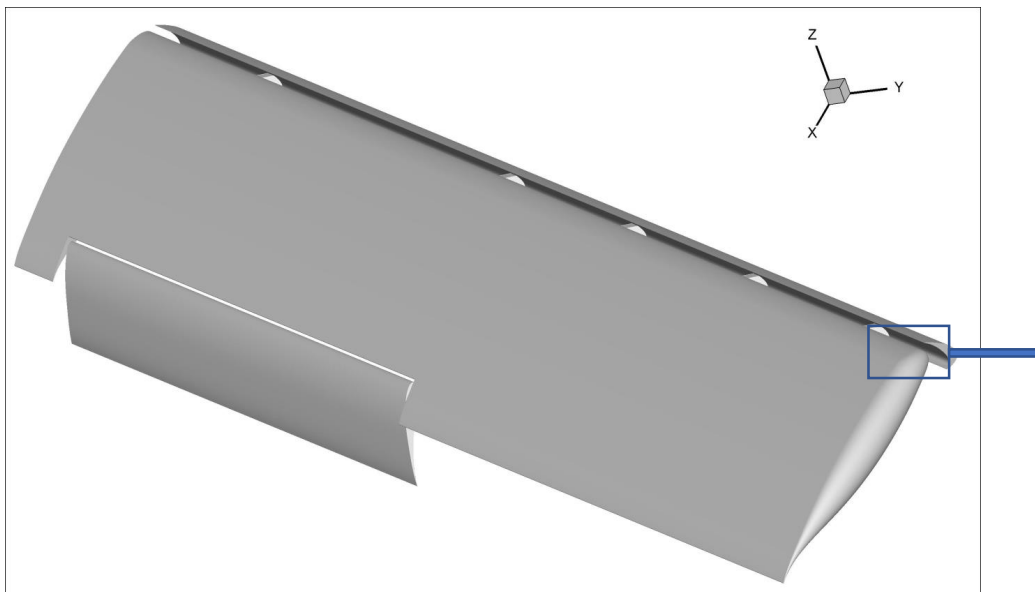




CAD Geometry and Mesh Issues



Tetrahedral elements whose four vertices all lie on the solid surface may occur at surface intersections. For a vertex-based finite-volume method, these elements lead to a singular system as the CFL number is increased toward infinity, particularly when using low-Mach-number inviscid flux schemes such as HLLC-LM.

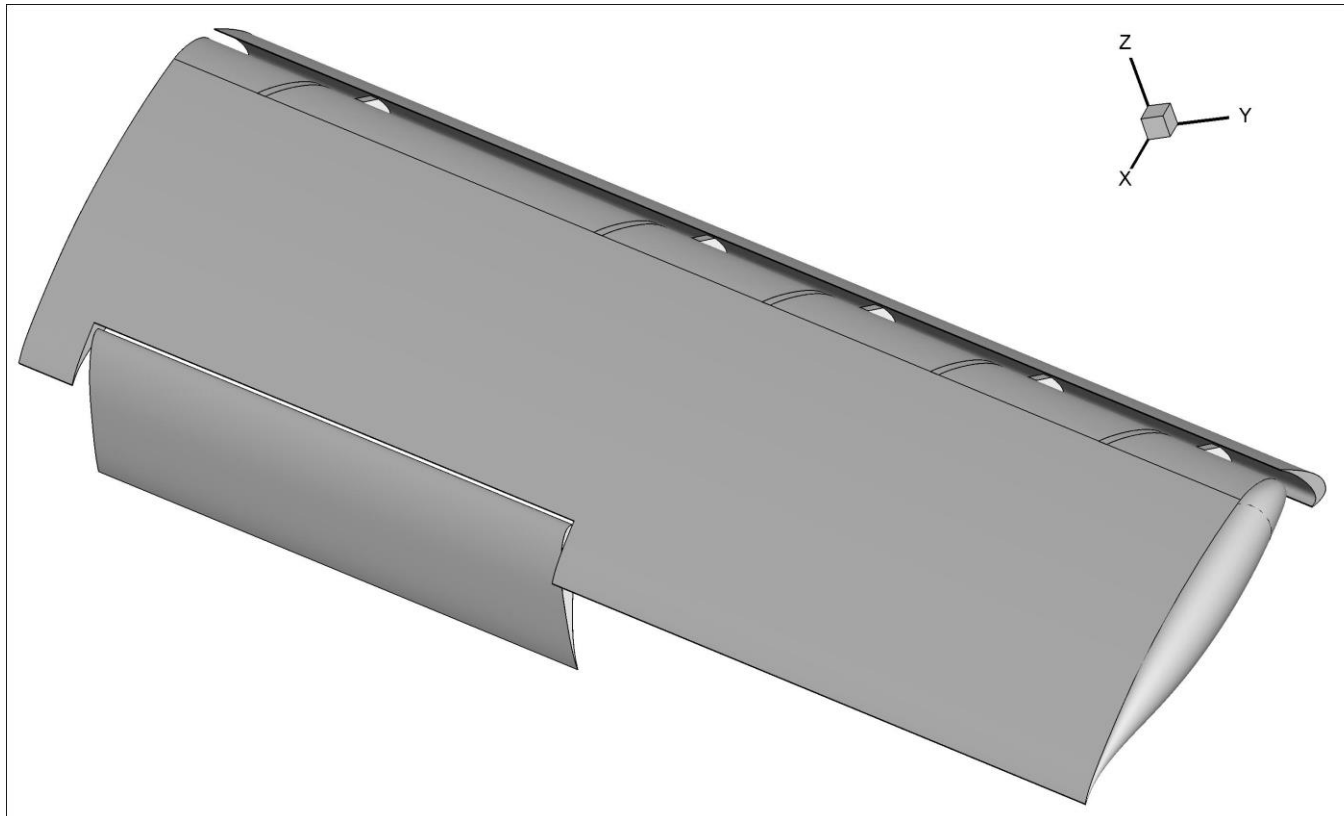


There are also extremely small triangular elements at main wing outboard section close to leading edge.



New CAD Model

Main wing CAD model has been improved at the tip and surfaces have been added downstream of the brackets to improve the resolution of the bracket wakes.

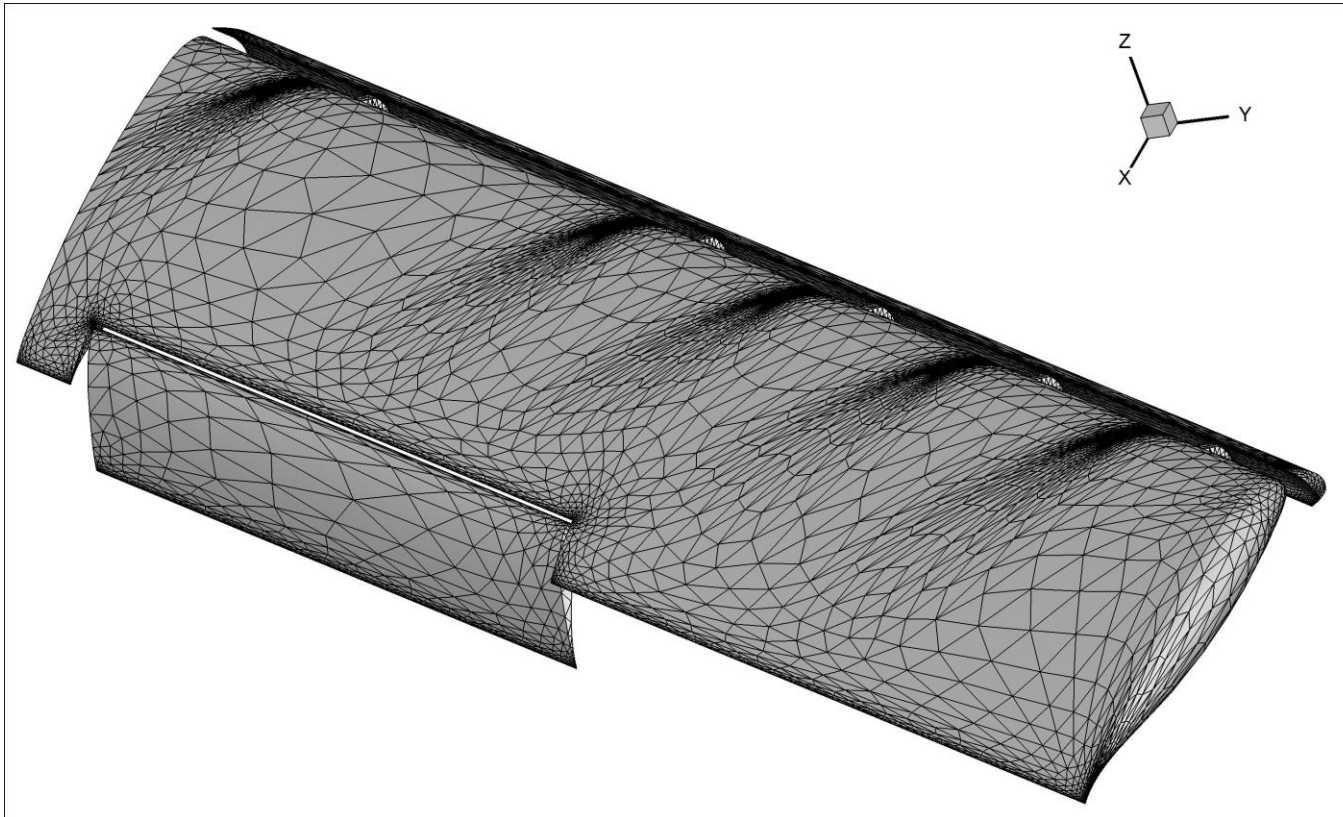




New Initial Mesh (Refine Bootstrap)

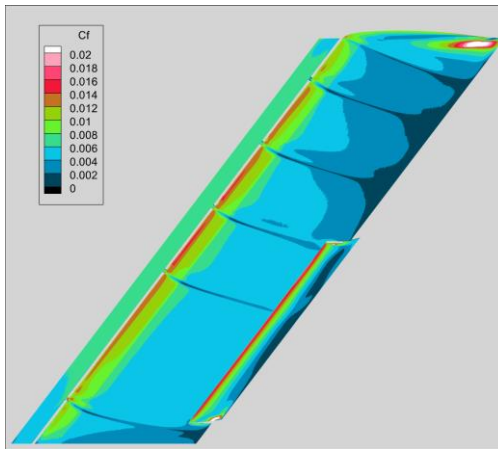
The mesh resolution in the bracket wake regions has been improved.

Tetrahedral elements with all vertices on the solid wall have been eliminated in the final NASA Refine version, thanks to Mike Park.

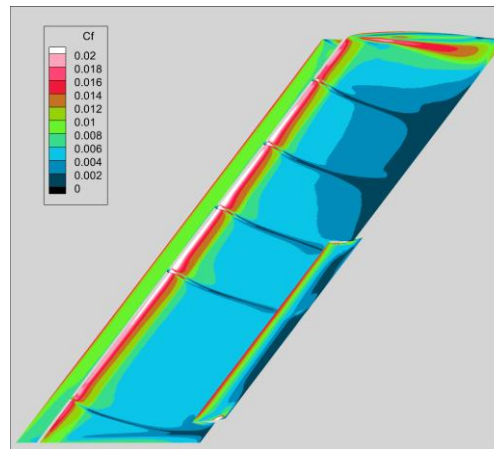




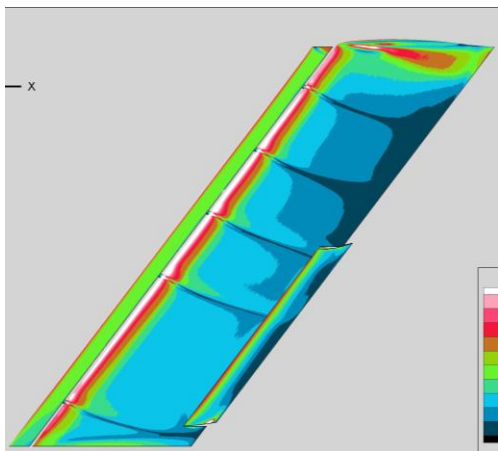
Surface Skin Friction Coefficients (Roe)



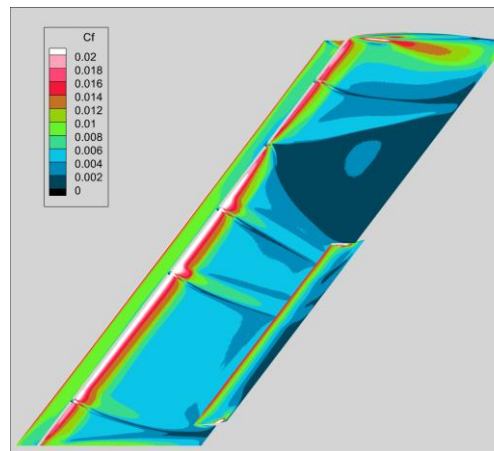
Alpha=10 (40M)



Alpha=20 (40M)



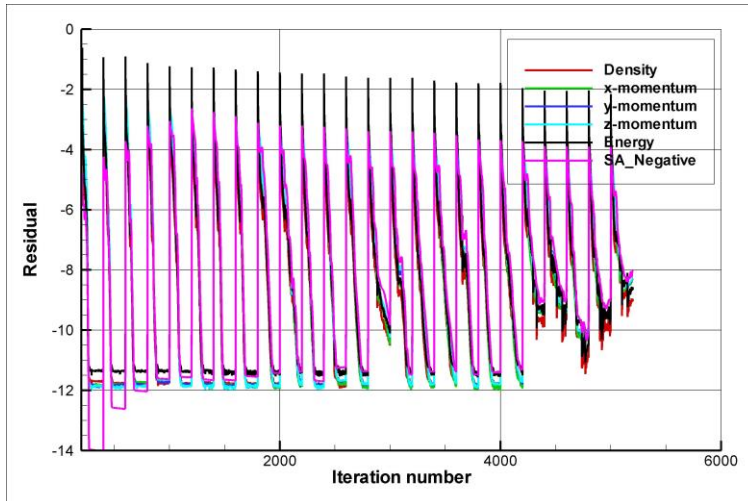
Alpha=23 (8.5M)



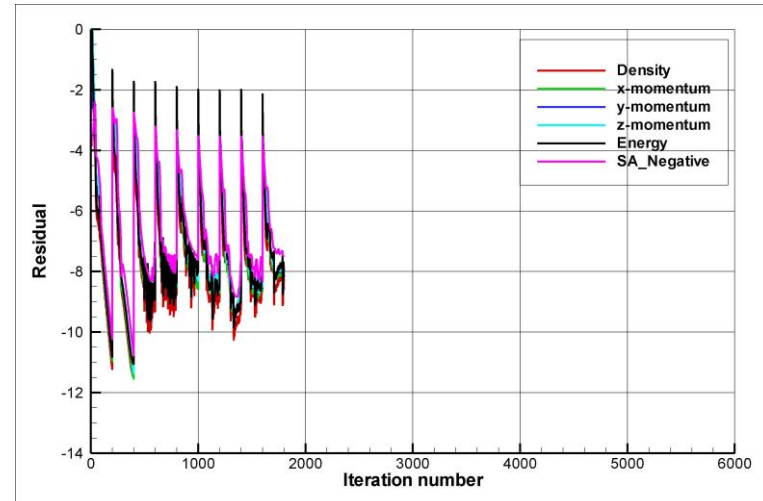
Alpha=25 (40M)



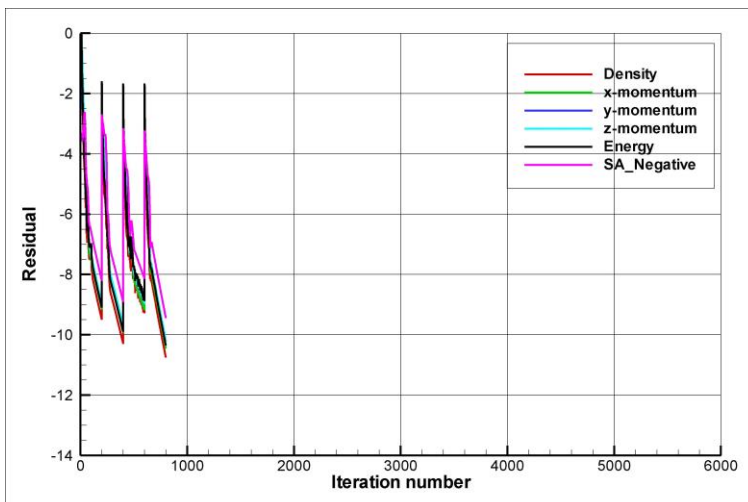
Residual Convergence with Mesh Refinement (Roe)



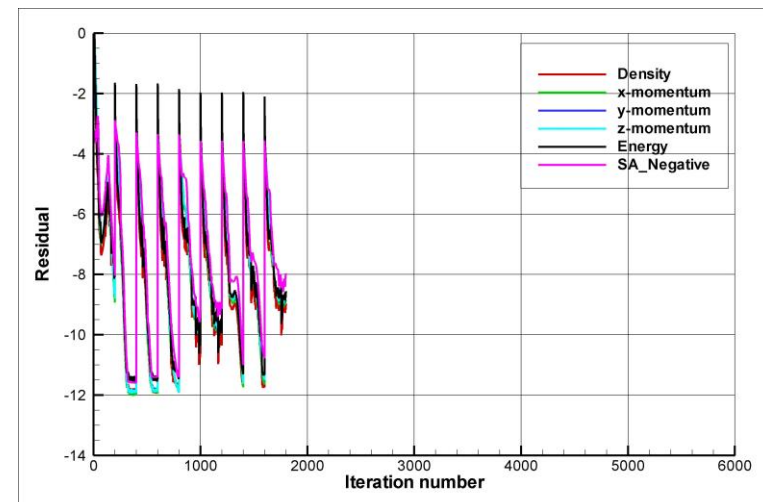
Alpha=10



Alpha=20



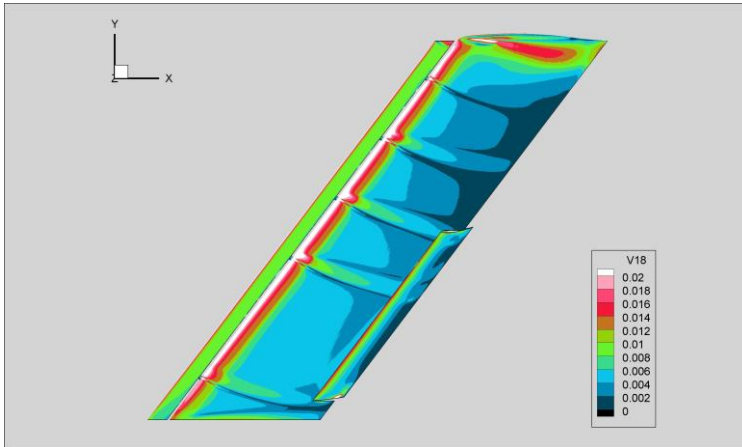
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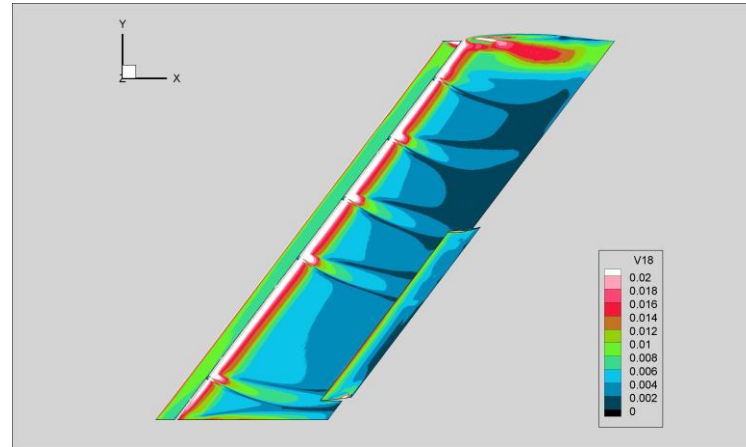
Alpha=25



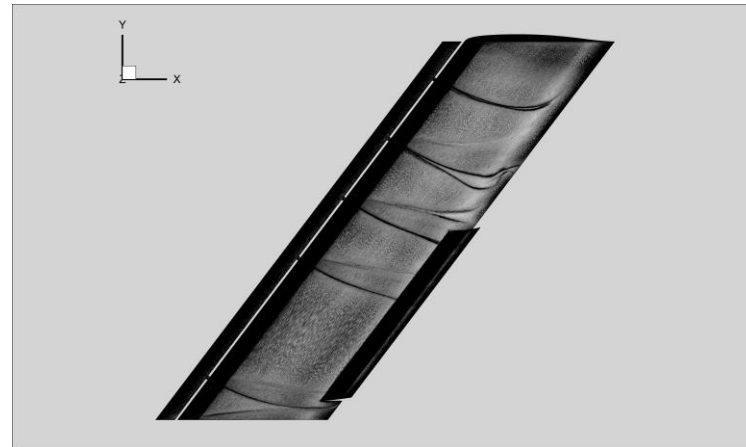
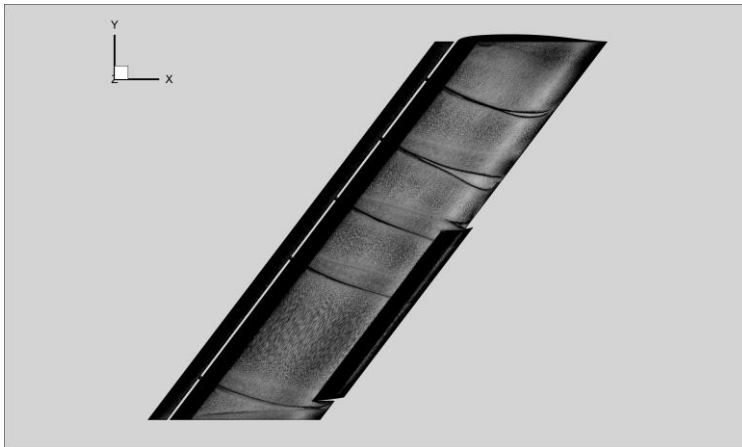
Surface Skin Friction Coefficients and Adapted Meshes (AUSMpM)



Alpha=25 (40M)



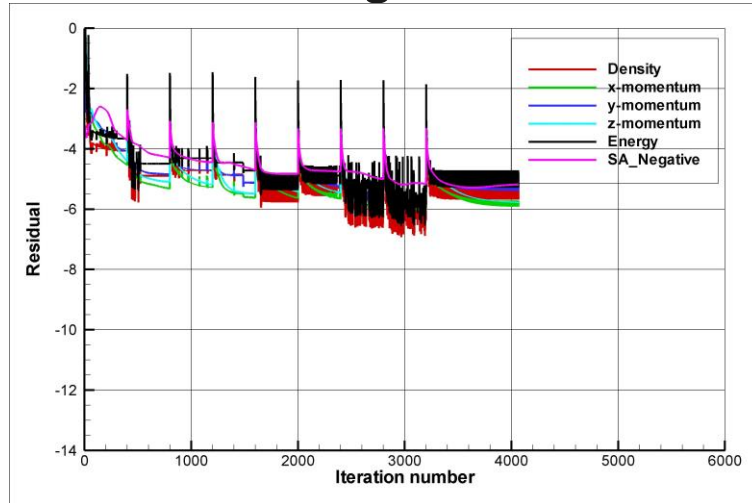
Alpha=30 (40M)



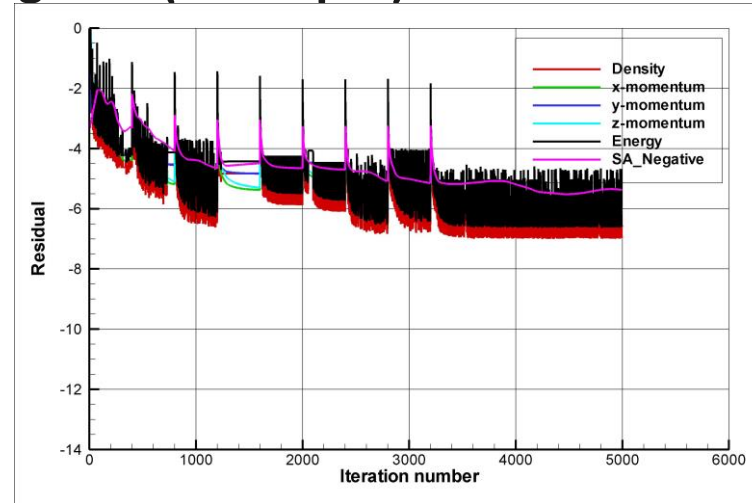
These results did not fully converged!



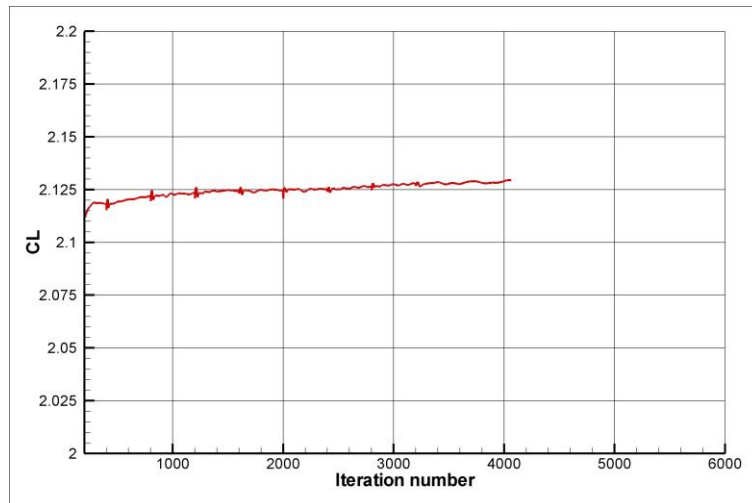
Residual Convergence and Lift Convergence (AUSMpM)



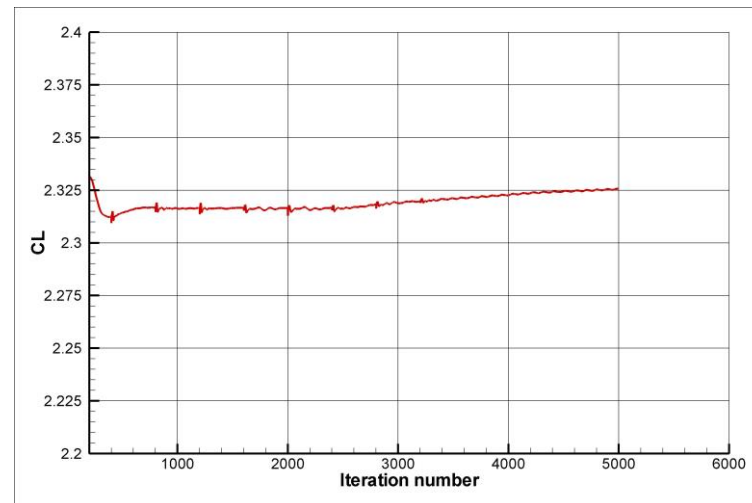
Alpha=25



Alpha=30



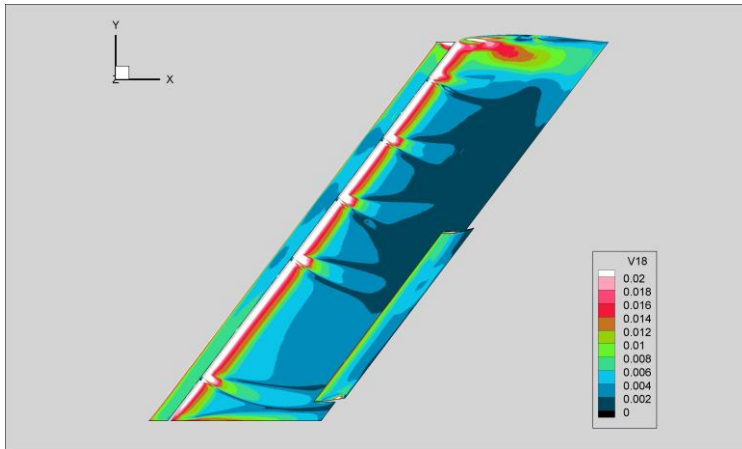
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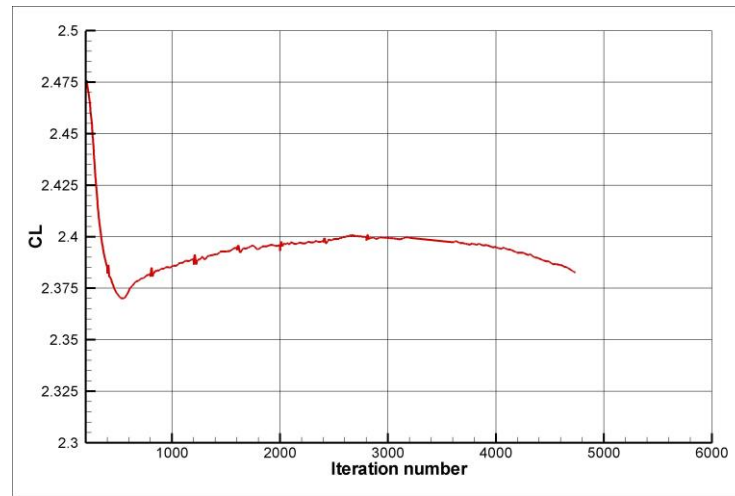
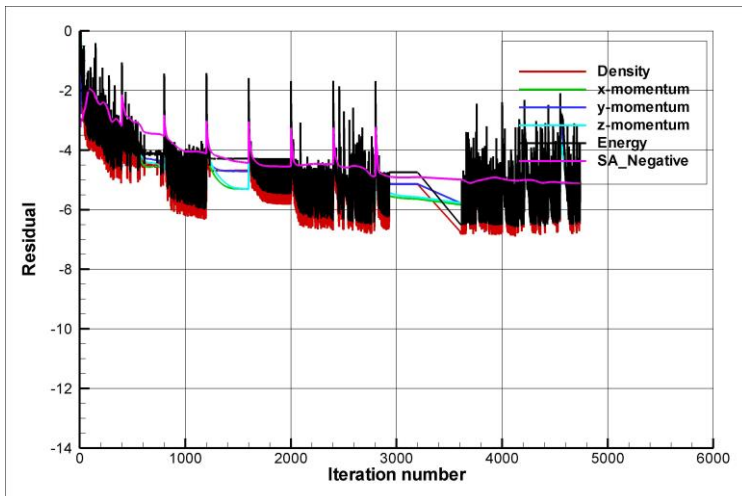
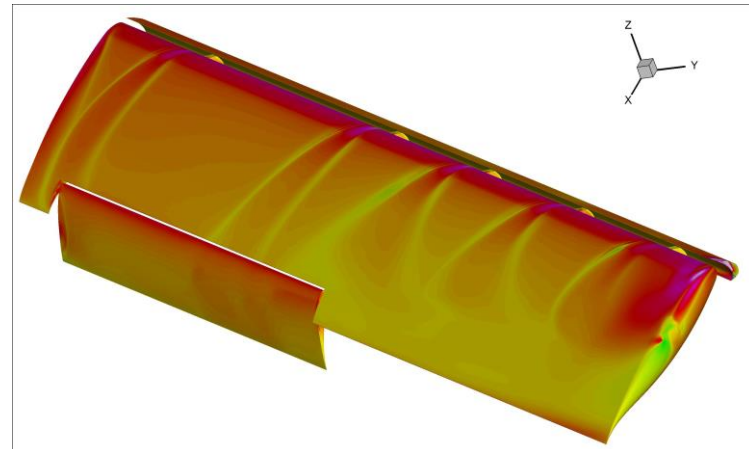
Alpha=30



Surface Skin Friction Coefficients and Adapted Meshes (AUSMpM)



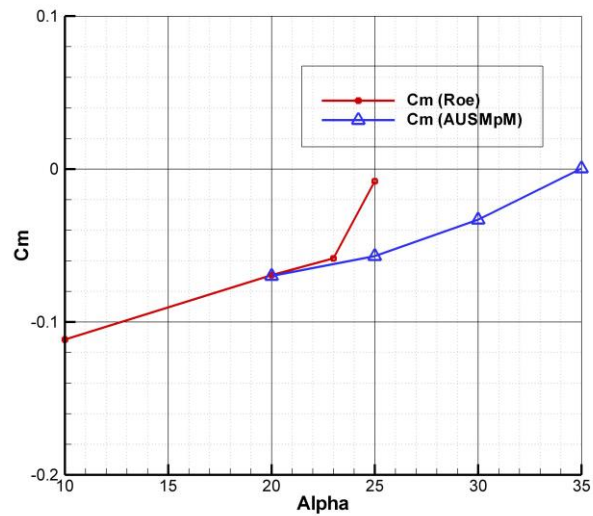
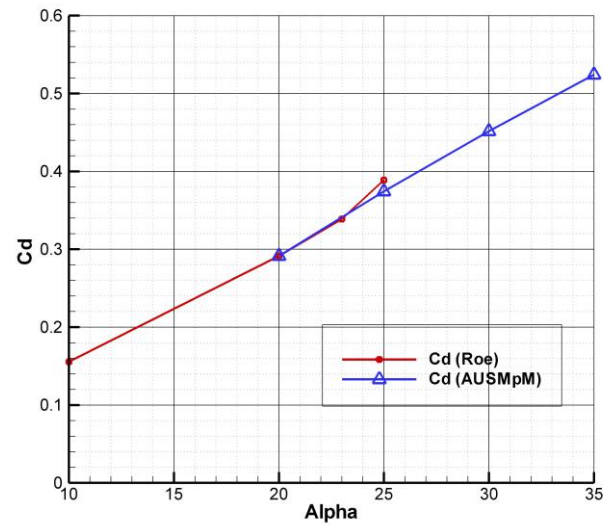
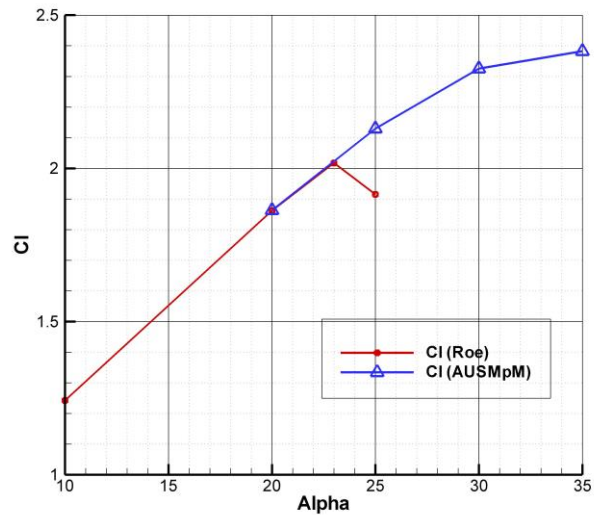
Alpha=35 (40M)



These results did not fully converged! Too expensive with 40M to continue for us unless high CFLnumber is achieved.

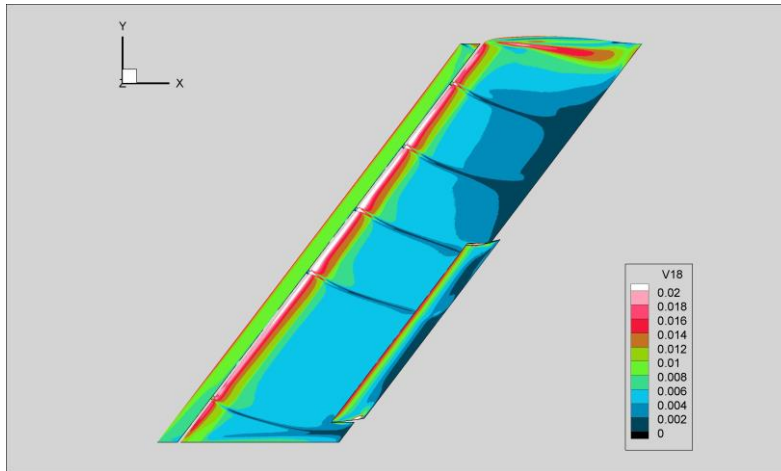


Variation of Aerodynamic Loads with Angle of Attack





Comparison of Surface Skin Friction Coefficients

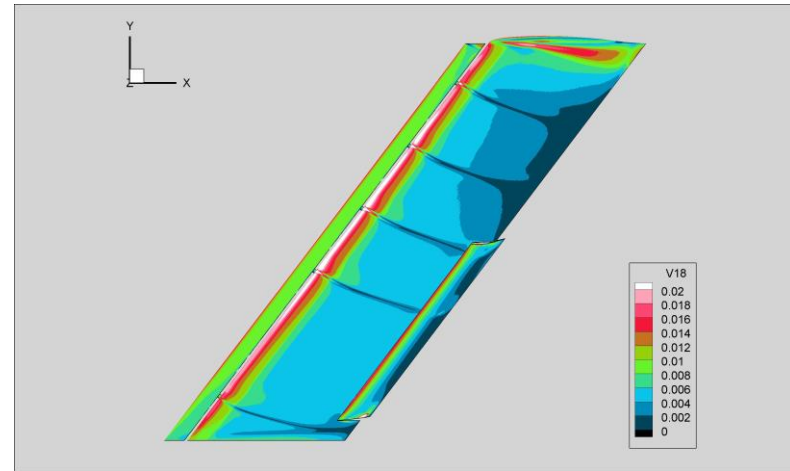


Alpha=20 with Roe (40M)

$C_d=0.291422890$

$C_l=1.8624057697$

$C_m=-0.0693903336$



Alpha=20 with AUSM+M (40M)

$C_d=0.2913975041$

$C_l=1.8631049170$

$C_m=-0.0699091060$



Numerical Issues Related to High Lift Simulations

These are issues considered:

- Vortex Resolution and Numerical Diffusion
- Turbulence
- Least-Squares Reconstruction
- Pseudo-Transient Continuation (PTC)
- Zero Eigenvalues
- ILU(k) Preconditioner



Vortex Resolution and Numerical Diffusion

- The first mechanism is associated with the horseshoe vortices generated at the upper and lower junctions of the bracket with the main wing and slat. The vortex originating from the main-wing junction is relatively stronger and passes over the leading edge, where the flow undergoes strong acceleration. As a result, the streamwise component of the horseshoe vortex is strongly stretched and amplified. However, the regions of sharp curvature near the leading edge are generally refined preferentially in the direction of strong curvature, resulting in highly anisotropic, high-aspect-ratio elements. Such elements may provide **insufficient transverse resolution of the vortex**, causing excessive numerical diffusion that shortens and weakens the coherent streamwise vortex. This can significantly alter the vortex–boundary-layer interaction and ultimately lead to premature separation. An isentropic mesh-adaptation strategy, as proposed by Aliaga et al., may provide a better alternative by improving the resolution of the relevant vortices.
- The use of the AUSM+M scheme may reduce numerical diffusion near solid walls, where the Mach number approaches zero, allowing the horseshoe vortices to remain coherent over a longer distance. This is particularly important because the numerical dissipation of many conventional flux schemes increases as the Mach number decreases (scales with $1/\text{Mach}$), whereas AUSM+M is designed to maintain a low-dissipation behavior in the low-Mach regime (scales with Mach). These stronger and more persistent vortices may enhance momentum transfer to the boundary layer and thereby help prevent premature separation. However, the residual convergence problem with AUSM+M still persists, and it is therefore not yet clear whether the flow will ultimately remain attached for 25° , 30° , and 35° or develop separation once a fully converged solution is obtained with large CFL numbers. However, the recent implementation of the Weiss–Smith low-Mach-number preconditioner led to a residual reduction of $1\text{d}–10$ with a maximum CFL number of 4570 on the coarse level (8M nodes), while maintaining attached flow at an angle of attack of 25 degrees for the TC1 where where classical Roe scheme led to separation at the same angle. **However, the convergence still deteriorates on fine levels.**
- At this point, the effect of the CFL number is not yet clear. Higher CFL numbers, as obtained with the Roe, HLLC, and AUSM+up schemes, consistently lead to separated flow, whereas the lower CFL numbers typically required by AUSM+M result in an attached flow, although the residual convergence eventually stalls/oscillates. It therefore remains unclear whether the observed difference in separation behavior is genuinely related to the high CFL number or is instead a consequence of the different numerical properties of the flux schemes. One possibility is that high CFL numbers, combined with a poorly conditioned Jacobian, may lead to inaccurate or unstable nonlinear updates that drive the solution toward premature separation. **This possibility requires further investigation on finer levels.**



Turbulence

- The second issue may be related to the strong coupling in the simultaneous solution of the RANS equations, where the turbulence-model residual can dominate the overall residual vector and the Jacobian entries associated with the SA coupling can become relatively large. This effect may be particularly important at high angles of attack, where the flow is strongly separated and the coupling between the mean-flow and turbulence equations becomes stronger.
- **The Jacobian verification performed with PETSc using finite differences showed some deterioration in the agreement between the analytical and finite-difference Jacobians when the SA model was activated.** However, calculations performed in quadruple precision indicated that the difference is on the order of 10^{-15} for a coarse mesh considered. This suggests that the observed discrepancy is likely related to round-off and finite-precision effects rather than an inconsistency in the analytical Jacobian. Furthermore, the use of the analytical second-order Jacobian instead of finite difference approximation did not significantly alter the results.
- Another potential source of inconsistency is that, in the current HEMLAB implementation, **the convective term in the SA equation is not compatible with the mass flux used in the Navier–Stokes inviscid flux formulation.** This inconsistency may introduce an artificial source in the turbulence convection term and could become more significant in the high-angle-of-attack regime.
- **The horseshoe vortices may also undergo excessive dissipation due to the relatively high turbulent viscosity predicted at their cores.** This can weaken the vortices and reduce their ability to transfer momentum to the boundary layer, thereby promoting premature separation. A rotation/curvature correction to the Spalart–Allmaras model (SA-R) may therefore be beneficial by reducing the turbulent viscosity in strongly rotating vortex cores and preserving the coherent horseshoe vortices. This could enhance their momentum transfer to the boundary layer and help delay or prevent premature separation.
- Another potential issue is the use of negative ν_{hat} values in the Spalart–Allmaras turbulence model. **In regions where the model predicts negative ν_{hat} , the turbulent viscosity is effectively driven toward zero, potentially even below those of the freestream. This may lead to local laminarization of the flow,** particularly in regions that are already prone to separation, and could therefore significantly affect the predicted separation behavior. This may also be important in regions where multiple vortices interact, particularly when the vortices are poorly resolved and their local turbulent-viscosity predictions become less reliable.



Least-Squares Reconstruction

- The third issue may be related to the least-squares interpolation used to construct the left and right states for the inviscid fluxes. Calculations for DPW-8 using mixed-element meshes revealed a potential instability: the initial residual decreases to approximately 10^{-7} , after which it begins to increase and eventually leads to NaN values. A similar residual behavior is observed in the high-angle-of-attack HLPW cases immediately before the appearance of the pizza-slice separation pattern, although in these cases the solution subsequently evolves toward a separated steady state.
- Interestingly, for DPW-8, **the instability is eliminated when the state vectors at solid-wall boundaries are constructed using a central approach rather than the one-sided least-squares reconstruction**. This observation suggests that the one-sided least-squares construction near solid walls may be problematic, potentially because the available stencil becomes highly asymmetric and poorly conditioned in the presence of mixed-element meshes, where generally only one node is located inside the fluid domain.
- **The same modification also improved the premature separation observed in HLPW5 Case 2.2, although no improvement was observed for Case 2.4 on the coarsest committee Cadence mixed-element mesh.** This difference suggests that the sensitivity to the least-squares reconstruction may depend strongly on the local flow topology and mesh characteristics. However, the same least-squares approach does produce attached flow with AUSM+M. The central approach used at the solid wall may help to reduce numerical dissipation and improve the accuracy of the low-Mach-number characteristics, leading to attached flow.



Pseudo-Transient Continuation (PTC)

- The use of pseudo-transient continuation (PTC) has been suggested by DLR as a potential contributor to the observed premature separation.
- The mechanism is related to the different characteristic time scales of the boundary layer and wake, which may be affected differently by the artificial pseudo-time evolution introduced by PTC. **In the boundary layer, PTC appears to produce artificially high vorticity, increasing the SA production term and consequently the turbulent viscosity. In the wake and boundary layer, artificial flow stretching may also increase the streamwise vorticity, which can similarly increase the SA production and turbulence viscosity.** The resulting excessive turbulent viscosity in both regions may enhance diffusion of the vortical structures, weaken the horseshoe vortices that energize the boundary layer, and thereby contribute to premature separation.
- The pseudo time limit proposed by DLR may also improve the diagonal dominance of Jacobian matrix and hence increases the stability of ILU(k) preconditioner.
- **One way to demonstrate/prove the effects of PTC algorithm is to implement adaptive time stepping (ATS) without PTC.** The PETSc library does support adaptive time stepping, and its application to the present problem is currently under investigation.



Zero Eigenvalues and ILU(k) Preconditioner

- The Jacobian matrix may exhibit zero or near-zero eigenvalues for several reasons. Such eigenvalues may indicate a physical bifurcation, where the solution branch is approaching a loss of local uniqueness, or may arise from global conservation properties in the finite-volume discretization and their interaction with the imposed Dirichlet boundary conditions. **In the presence of a zero or near-zero eigenvalue, the Newton update can become very large in the corresponding eigenvector direction, since the Jacobian provides little resistance to perturbations along this mode.** Pseudo-transient continuation can alleviate this issue by shifting the eigenvalue spectrum away from zero. However, as the CFL number increases, the pseudo-transient contribution becomes progressively weaker. Therefore, the eigenvalues closest to zero should be tracked throughout the alpha sweep to determine whether the onset of premature separation is associated with a near-singular Jacobian.
- SLEPc library works directly with PETSc and can compute the leading eigenvalues/eigenvectors (CAF 2025, Sukas&Sahin).
- The calculations are generally based on ILU(k)-type preconditioners, and their stability at high CFL numbers remains an open question. **The stability of the ILU factorization can be particularly sensitive to small entries in the Jacobian matrix,** which may become increasingly important as the CFL number is increased. In particular, the vanishing or nearly zero diagonal entry associated with the continuity equation requires special consideration as the CFL approaches infinity, especially when no additional stabilization is employed.



Conclusions for TC1

- The Roe scheme fails to predict the attached-flow pattern at $\text{Alpha}=25$ degrees, despite achieving machine-precision convergence with the PETSc SNES nonlinear coupled solver. The solver converges robustly to a steady-state solution, even in cases involving large-scale flow separation.
- In contrast, **the AUSM+M scheme introduces substantially less numerical dissipation, with dissipation that scales with Mach number** rather than inversely with Mach number, as observed for the Roe scheme. Consequently, AUSM+M successfully avoids the premature leading-edge separation observed with Roe. However, the scheme is unable to sustain high CFL numbers and does not converge to a steady-state solution, indicating that further work is required to improve its nonlinear convergence and robustness.
- Although several factors may contribute to the premature flow separation—including inadequate mesh resolution of the horseshoe vortices generated at the upper and lower junctions of the bracket, turbulence modeling that may produce excessive turbulent viscosity, issues associated with least-squares gradient reconstruction near solid walls, pseudo-transient continuation (PTC), zero Jacobian eigenvalues as the solution branch approaches a loss of local uniqueness or becomes linearly dependent due to Dirichlet boundary conditions, and ILU(k) preconditioning in the presence of very small matrix entries—**it is likely that these issues are manifestations of a single underlying numerical problem.**
- A brute-force increase in mesh resolution would be very expensive for RANS unless the critical issues identified above are addressed first, particularly in regions where mesh resolution and mesh quality are most critical.



Acknowledgements

EuroHPC – etur112

Access to MareNostrum 5 supercomputer at Barcelona Supercomputing Center (BSC), Spain

UYBHM

National Center for High Performance Computing of Turkey (UYBHM) under grant numbers 1026862026 and 5014832023.



Test Case 1 (Fortran nml Input File)

```
&ADAP_OPTS
  adap_sizes = (200000 ; 400000 ; 1000000 ; 2000000 ; 4000000 ; 10000000 ; 20000000 ; 40000000 ;
100000000),
  adap_norm = 4,
  adap_subite = (4 ; 4 ; 4 ; 4 ; 4 ; 4 ; 4 ; 4),
  adap_ext_ite = (400 ; 400 ; 400 ; 400 ; 400 ; 400 ; 400 ; 400 ; 400 ; 400),
  adap_residual_reduction = (3 ; 3 ; 3 ; 3 ; 3 ; 3 ; 3 ; 3),
  adap_sensor = (MACH; MACH; MACH; MACH; MACH; MACH; MACH; MACH; MACH; MACH),
  adap_hmin = 1e-6,
  adap_hmax = 100.,
  adap_hgrad = (-10 ; -10 ; -10 ; -10 ; -10 ; -10 ; -10 ; -10 ; -10),
  adap_aspect_ratio= -1000000,
  adap_exit_iteration = 25,
  adap_spalding = 0.000001,
  adap_bl_exit_iteration = 25,
  adap_viscous_tags
=(7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;30;31;32;33;34;35;36;37;38;39;40;41;42;
43;44;45;46;47;48;49;50;51;52;53;54;55;56;57;58;59;60;61;62;63;64;65;66;67;68;69;70;71;72;73;74;75;76;77;7
8;79;80;81;82;83;84;85;86;87;88;89;90;91;92;93;94;95;96;97;98;99;100;101;102;103;104;105;106;107;108;109;
110;111;112;113;114;115;116;117;118;119;120;121;122;123),
  bl_creation= false,
  delete_files= false,
  additional_refine_metric_options= "" ,
  additional_refine_adap_options= "" ,
  additional_refine_adap_bl_options= «»,
  adap_back_file= "ite20.egads",
/
```

```
&SOLVER_OPTS
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  mesh_file = "ite20.mesh",
  mesh_scale = 1.0d0,
  restart_file = "ite20_7400.dat",
  spatial_num_order = "second_order", !"third_order", !"second_order",
  spatial_turb_order = "second_order",
  spatial_beta = 0.333333d0,! 0.020833d0,
  spatial_beta_turb = 0.333333d0,
  conv_flux_scheme = "AUSMpM_t",
  problem = "Navier-Stokes", !"Euler",!
  turb_model = "RANS-SA",!none
  SA_ft2= T,
  SA_R= F,
  Riemann_BC = T,
  Re = 3.55d0,
  Re_refL = 1d0,
  Area_ref = 2.4d0,
  Moment_ref_loc = 1.160273333d0, 0d0, 0d0,
  Pr = 0.72d0,
  Pr_t = .9d0,
  mu = 1d-6, ! non-dimensional viscosity
  nu_0 = 3.0d0, ! ratio to mu value 3.0 SU2 value?
```

```
dual_time = T,
dt = 100.d10,
CFL_param = 10.0d0, 10000d0, 5d0, 1d9, 1.125d0, 10d0, 1d-7, 1d0,
CFL_adap = T,
CFL_dist = F,
CFL_lmin = F,
alpha = 30d0,
sideslip = 0.0d0,
M_infty = 0.2,
rho_i = 1d0,
p_i = 0.714285714286d0,
T_i = 1.0d0,
Suther_temp_over_Free_temp = 0.38315d0, ! Default 110.4d0 K /273.15d0 K
gammaVal = 1.4d0,
auxillary = F,
implicit = T,
newton_ite = 1,
newton_omega = 1d0,
exit_ite = 200,
write_interval = 1000000,
print_interval = 1,
Limiter = F,
K_v = 0.001d0,
Eps = 0.001d0,
harten = 3
/
```

```
&BOUNDARY_INFO
1 = farfield,
2 = farfield,
3 = farfield,
4 = farfield,
5 = farfield,
6 = symmetry,
7 = noslip,
....
```