Iterative Convergence Criteria (ICC)

- Remarks on ICC
  - ICC are based on iterative convergence of aerodynamic coefficients
  - Residual convergence is monitored but not used in ICC
    - ✓ Sufficient residual convergence is hard to define
    - ✓ Residual convergence alone (e.g., X orders of magnitude reduction or residual norm less than Y) does not guarantee that the solution is close to the true solution on the given grid
  - ICC are not to grade solutions; they are to help participants to decide if more iterations are needed
    - ✓ Satisfying ICC does not guarantee that solution has sufficiently converged; it indicates that additional iterations are not likely to improve convergence
    - ✓ Conversely, failing ICC does not imply that solution is bad; it just indicates that additional iterations can be helpful
    - ✓ All solutions will be accepted, including those that do not satisfy ICC
- Two types of solutions satisfying ICC:
  1. Steady-state solution
  2. Solution with established oscillations

ICC for Steady-State Solutions

- All iterations mapped on [0, 1] interval
- Last 20%, 10%, and 5% of iterations considered
- Mean, slope, and standard deviation are computed on each interval
- ICC criteria:
  - Variation of mean between three intervals less than 1%
  - Slope convergence
    - ✓ Essentially zero slope computed over last 20% of iterations
    - ✓ Slope over last 5% is less than slope over last 10%, which is less than slope over last 20%
  - **Standard deviation on 20% interval less than 1%**

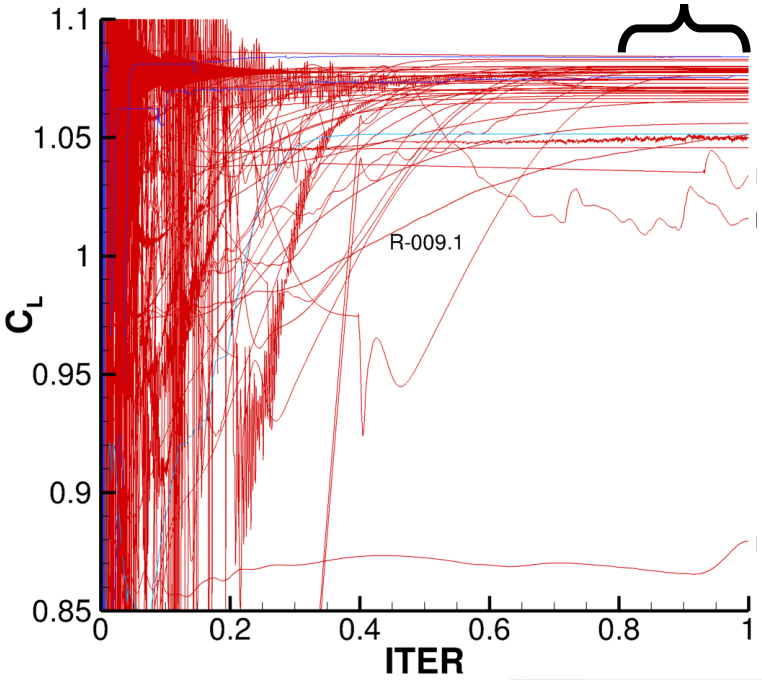
ICC for Solutions with Established Oscillations

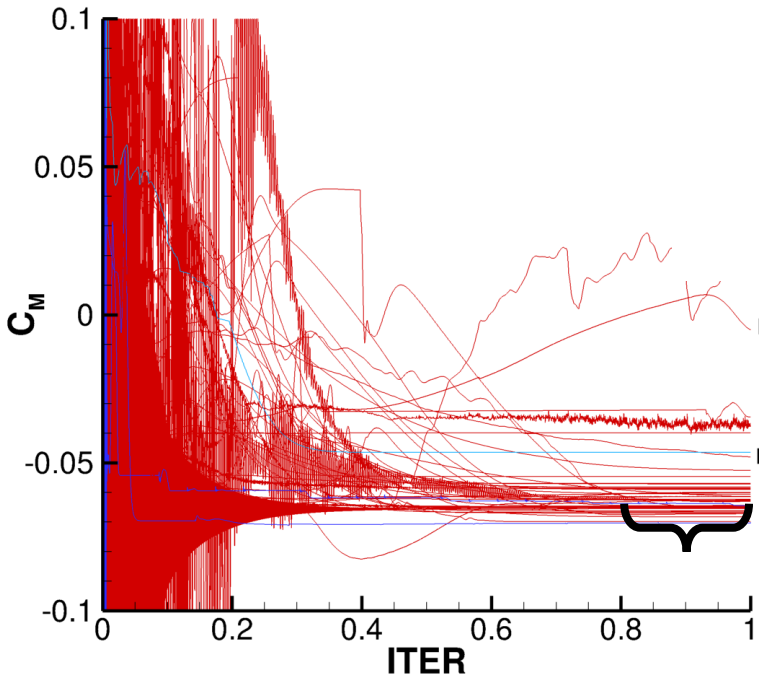
- All iterations mapped on [0, 1] interval
- Last 20%, 10%, and 5% of iterations considered
- Mean, slope, and standard deviation are computed on each interval
- ICC criteria:
  - Variation of mean between three intervals less than 1%
  - Slope convergence
    - ✓ Essentially zero slope computed over last 20% of iterations
    - ✓ Slope over last 5% is less than slope over last 10%, which is less than slope over last 20%
  - **Variation of standard deviation between three intervals less than 1%**

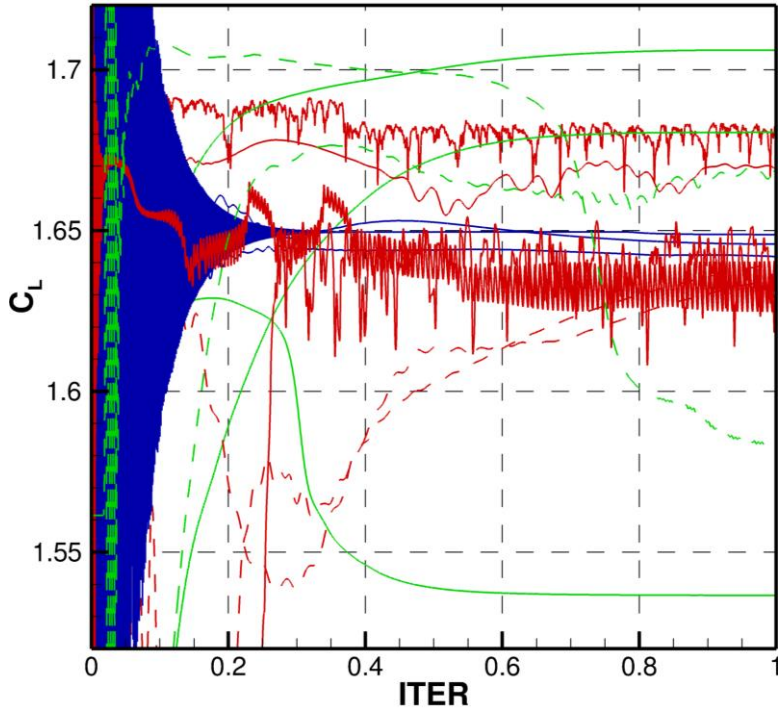
Grid Convergence Criteria (GCC)

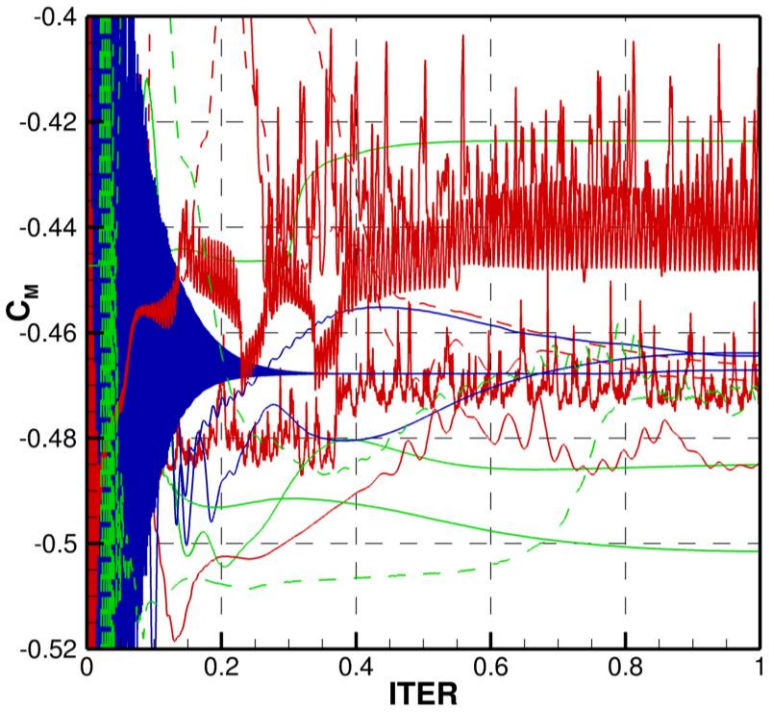
- Solutions on three or more grids in family, including fine enough grids
- Smaller variation between solutions on finer grids
- Aerodynamic coefficients plotted vs characteristic mesh size $\mathrm{h} = \mathrm{N}^{(-1/3)}$
  - N is degrees of freedom: nodes for node-centered and cells for cell-centered solutions
  - Flattening is expected for 2nd (and higher) order solutions

Iterative Convergence Illustrations



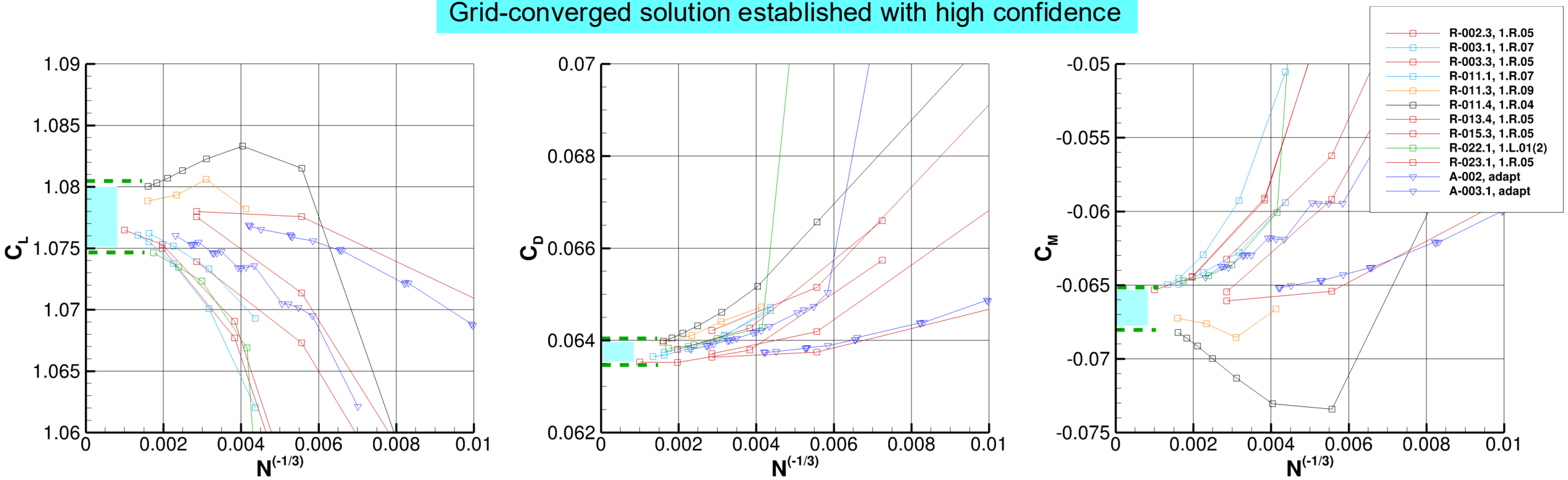






GCC for HLPW-5 Case 1, SA Solutions

Grid-converged solution established with high confidence



All SA solutions (shown for reference)

C_L: 0.9% range [1.075, 1.085]
C_D: 4% range [0.0635, 0.0660] (25 counts)
C_M: 9% range [-0.070, -0.064]

- Grid-convergence plots colored by grid family

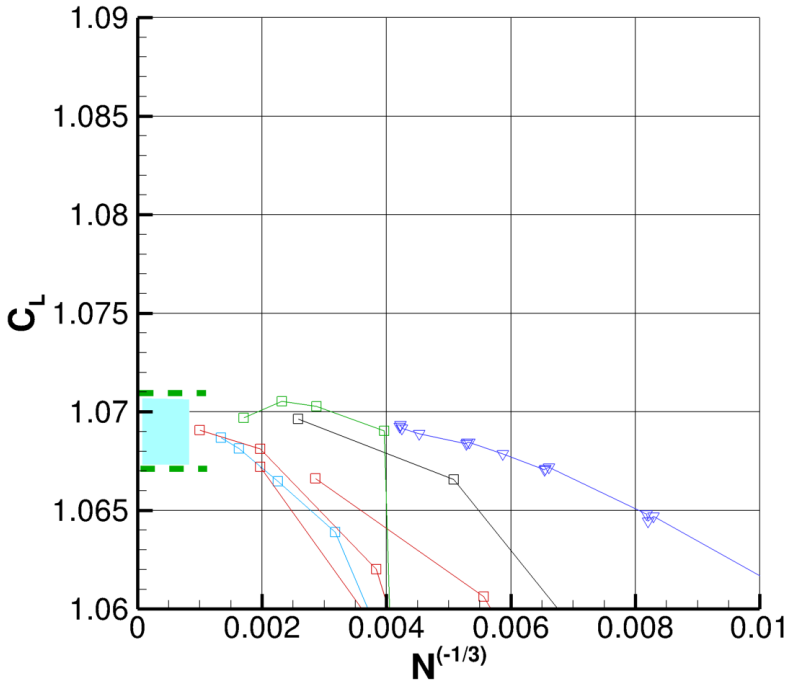
Selected SA solutions

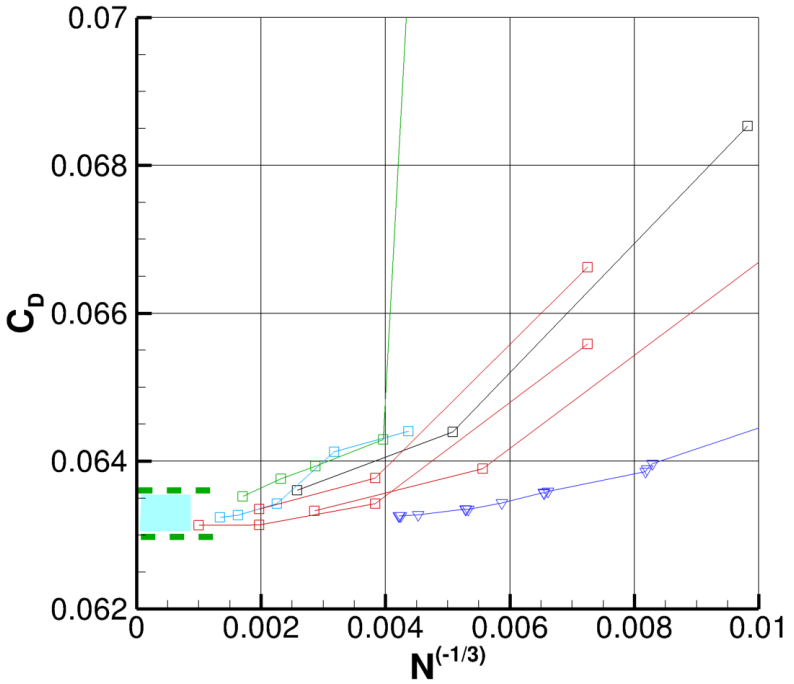
C_L: 0.45% range [1.075, 1.080]
C_D: 0.8% range [0.0635, 0.0640], (5 counts)
C_M: 4.5% range [-0.068, -0.065]

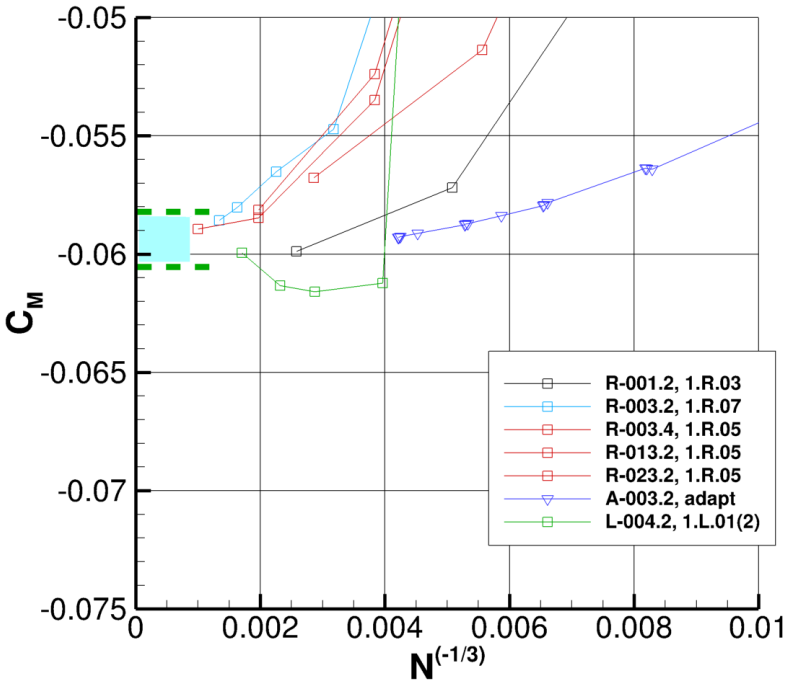
- Ranges of C_L and C_M reduced by factor 2
- Range of C_D reduced by 20 counts
- Residuals are well converged

GCC for HLPW-5 Case 1 SA-R(c_{rot}=1)-QCR2000 Solutions

Grid-converged solution established with high confidence







- Ranges for C_L and C_M in SA and SA-R(c_{rot}=1)-QCR2000 solutions do not overlap
- Distinctly different grid-converged solutions for each model
- Grid-convergence plots colored by grid family

All SA-R(c_{rot}=1)-QCR2000 solutions

C_L: 2.3% range [1.066, 1.081]
C_D: 10% range [0.0630, 0.0694] (64 counts)
C_M: 12% range [-0.063, -0.056]

Selected SA-R(c_{rot}=1)-QCR2000 solutions

C_L: 0.4% range [1.067, 1.071]
C_D: 1% range [0.0630, 0.0636], (6 counts)
C_M: 5% range [-0.061, -0.058]