Residual- and grid-converged multiple solutions for HLPW-5/HLPW-6 test cases with slat-bracket separation

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25 February 2026

General approach

RANS SA-neg-noft2 model

- Flow conditions from HLPW-5 website

SUPG finite-element discretization, 2nd order

Pseudo-transient continuation to steady state

- Fully differentiated residuals
- Machine-zero residual convergence

Goal-oriented adaptation (drag coefficient, pitching moment)

- A few fixed grid results (HELDENMESH, mixed elements) also presented
- Tet-only grids for the solution-adaptive approach

HLPW-5 Case 2.2 and HLPW-6 Case 1

GGNS T1/EPIC adaptive flow solver

Adaptation and α-continuation

Adaptive runs typically start from an inviscid *initial grid* and freestream “scratch” initial guess

- EPIC produces close to optimal adaptive grids driven by an anisotropic metric and a constraint on grid size
- The metric stems from a variant of anisotropic DWR error estimate [1]
- Adaptation is done for a sequence of prescribed *grid complexities*, typically, with multiple solve/adapt iterations at each fixed complexity level
- Interpolated converged solution from the previous adaptation level is always used as initial guess for Ψ-TC strong solver
- α-continuation is coupled with adaptation process
  - *is done on grid adaptation levels with “sufficient resolution” not on initial grid*

[1] Dmitry S. Kamenetskiy, Joshua A. Krakos, Todd R. Michal, Francesco Clerici, Frederic Alauzet, Adrien Loseille, Michael A. Park, Stephen L. Wood, Aravind Balan and Marshall C. Galbraith, “Anisotropic Goal-Based Mesh Adaptation Metric Clarification and Development”, AIAA SCITECH 2022 Forum, AIAA 2022-1245

Adaptation and α-continuation (contd.)

Inviscid initial grid, from-scratch solves were used as a major mechanism to trigger "spurious" Λ-separation ("Pizza Slice", PS) effect

- Exposes spurious component to the Λ-separation phenomenon
  - Starting from lower α's increases chances to get attached solutions
  - Starting from different high α's provides a variety of separated solution

α-continuation coupled with adaptation was used as a homotopy method

- Exposes regular component to the Λ-separation phenomenon

HLPW Case 2.2

α={10°,12°,13°,14°,15°,16°,16.5°}

- In red are angles alternative initial grid solves were attempted
- 16.5° is where most of the results are being presented

Adaptation *schedule:* target *grid complexities* (number of cells) and *multiplicities*

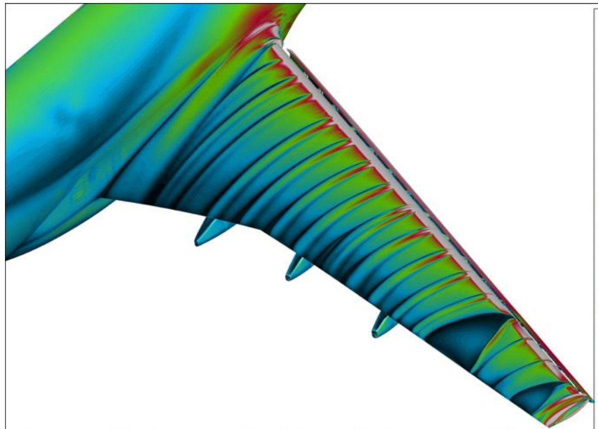
- *ADAPT_SpecifiedSize = 3x1e6 3x2e6 3x4e6 3x8e6 3x16e6 3x32e6 3x64e6 3x128e6 3x256e6 3x512e6*

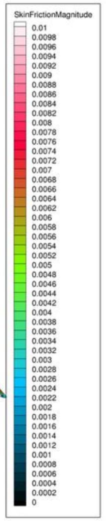
α-continuation was performed at the complexity level of 8M cells (~1.5M nodes)

- doing continuation on finer resolution levels was attempted, but did not lead to any different behavior

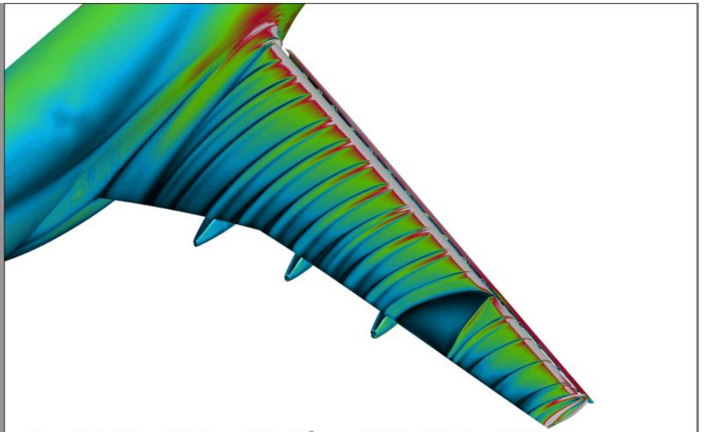
Finest adaptive grids had ~40M grid points

HLPW Case 2.2, 16.5 degrees

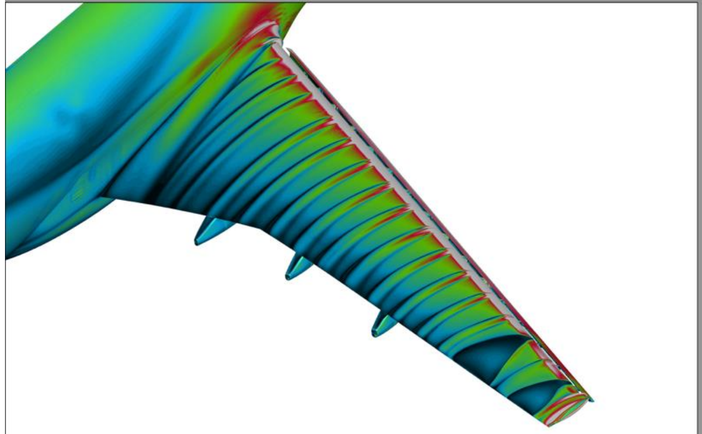




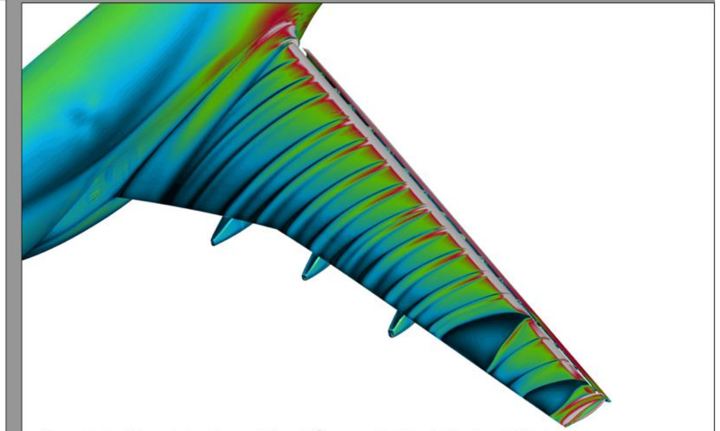
Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-1
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2



Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-3
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2

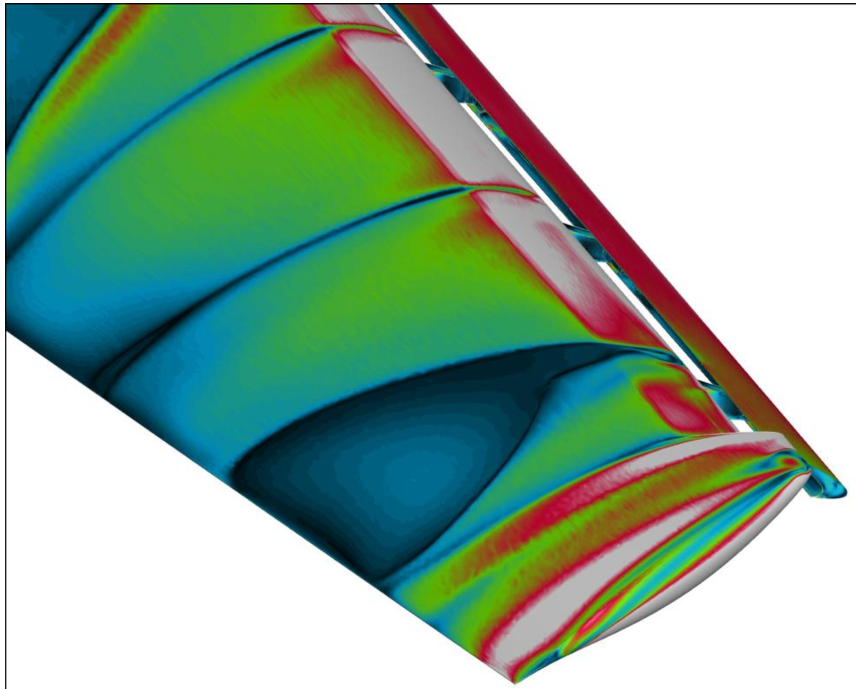


Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-2
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2

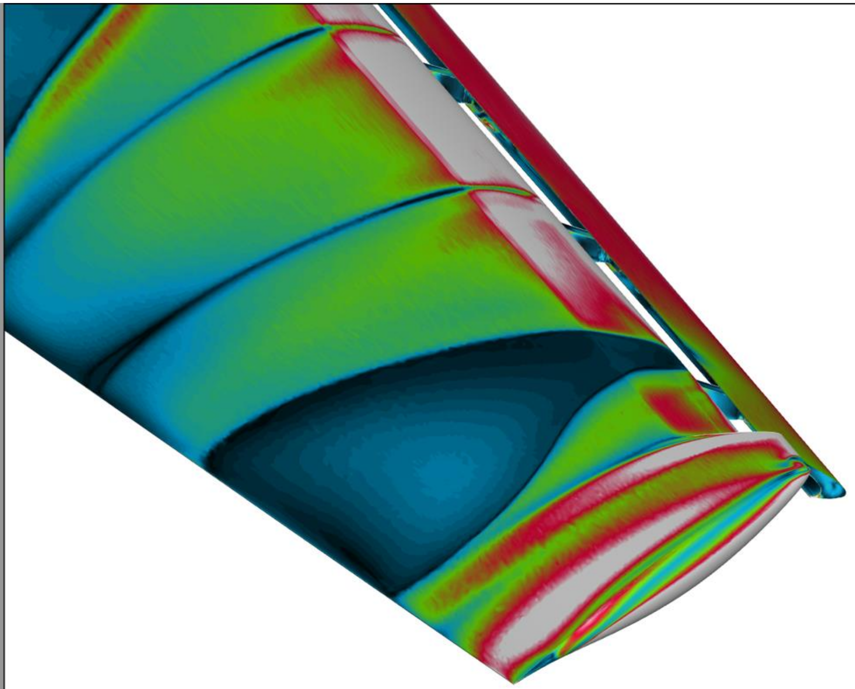


Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-4
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2

Different separation pattern at the same slat bracket

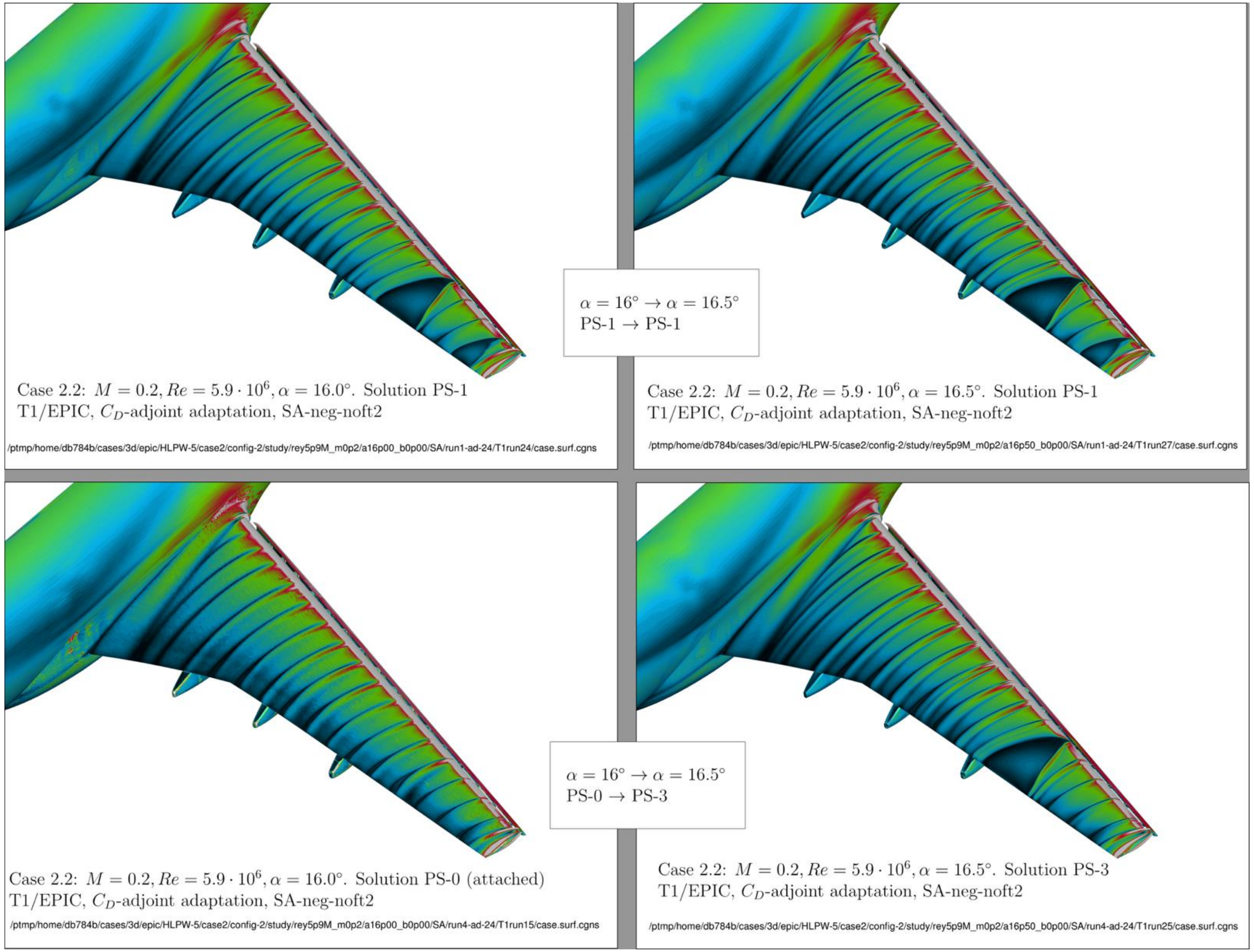


Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-1
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2



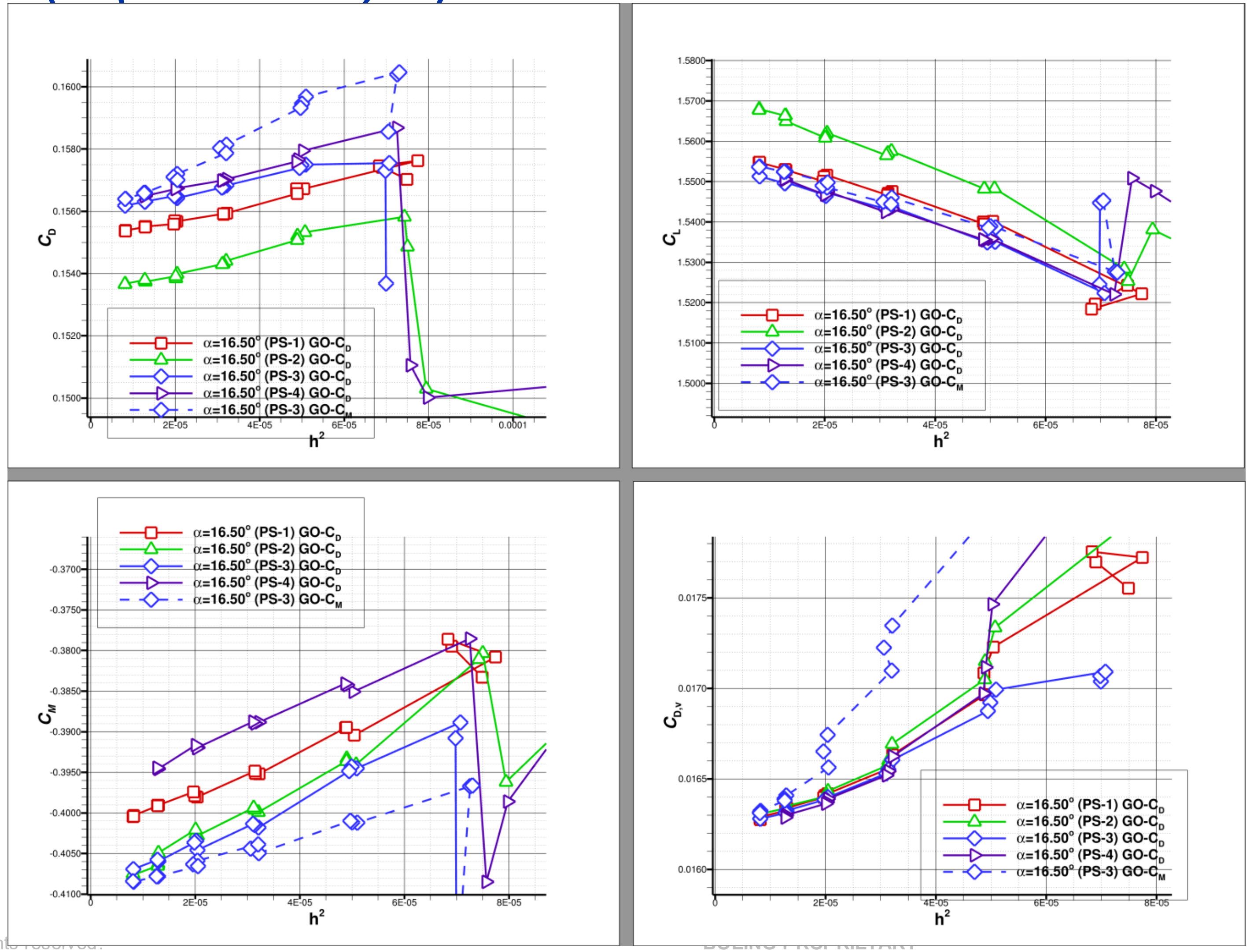
Case 2.2: $M = 0.2, Re = 5.9 \cdot 10^6, \alpha = 16.5°$. Solution PS-4
T1/EPIC, C_D-adjoint adaptation, SA-neg-noft2

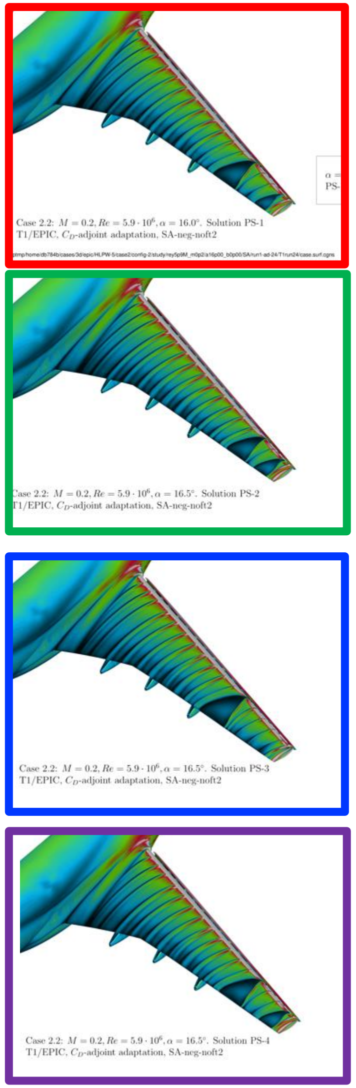
Regular vs. irregular behavior with α-continuation



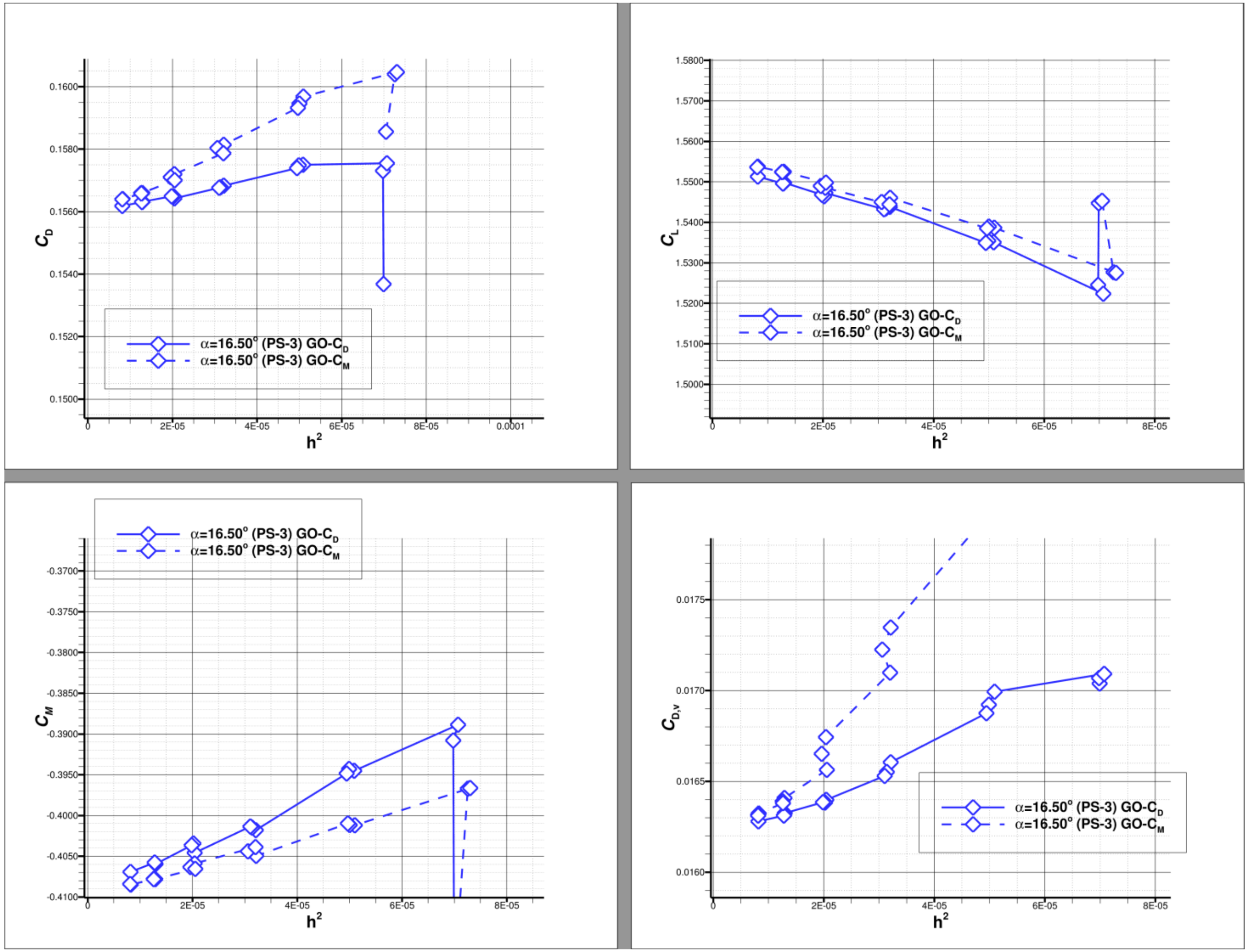
Force/moment grid convergence. Plotted against h^2

Increasing confidence in the $h \to 0$ solutions *(each of them)* by capturing the asymptotic-order slope (h=(# of nodes)$^{-1/3}$)



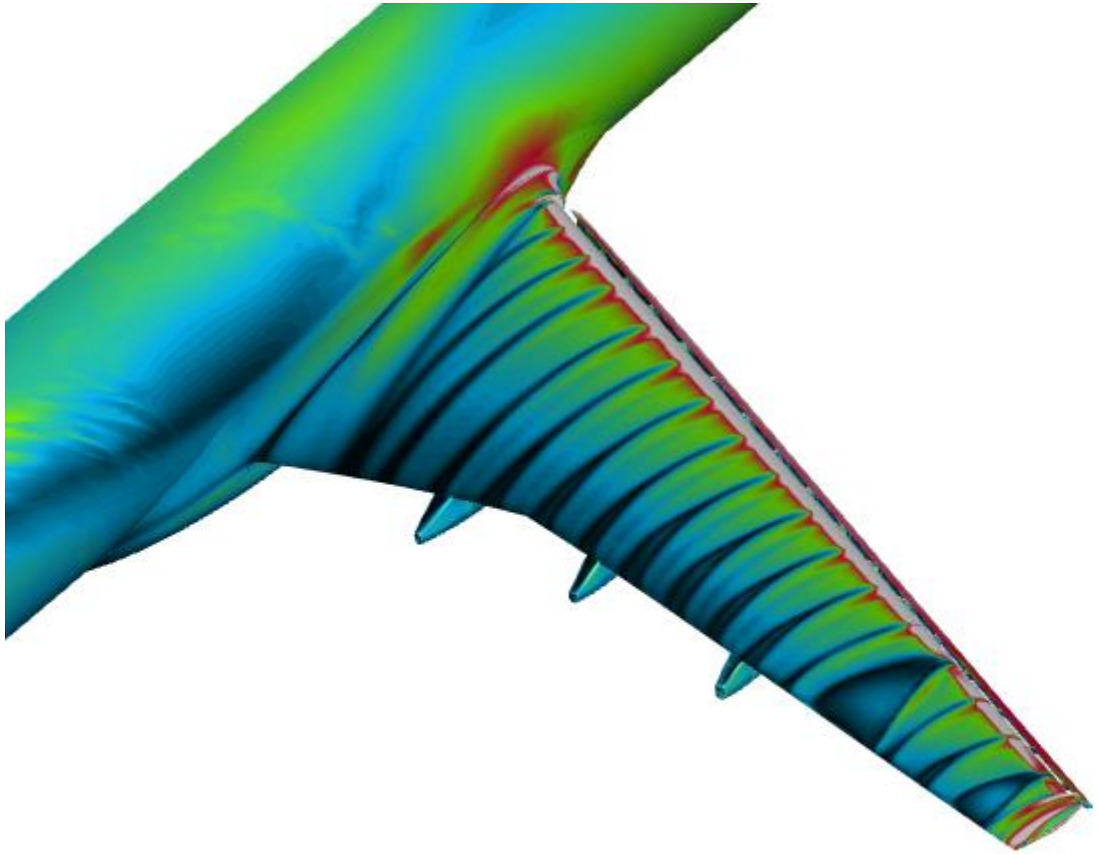


Increasing confidence in the $h \to 0$ solutions by capturing (in)sensitivity to the error estimate

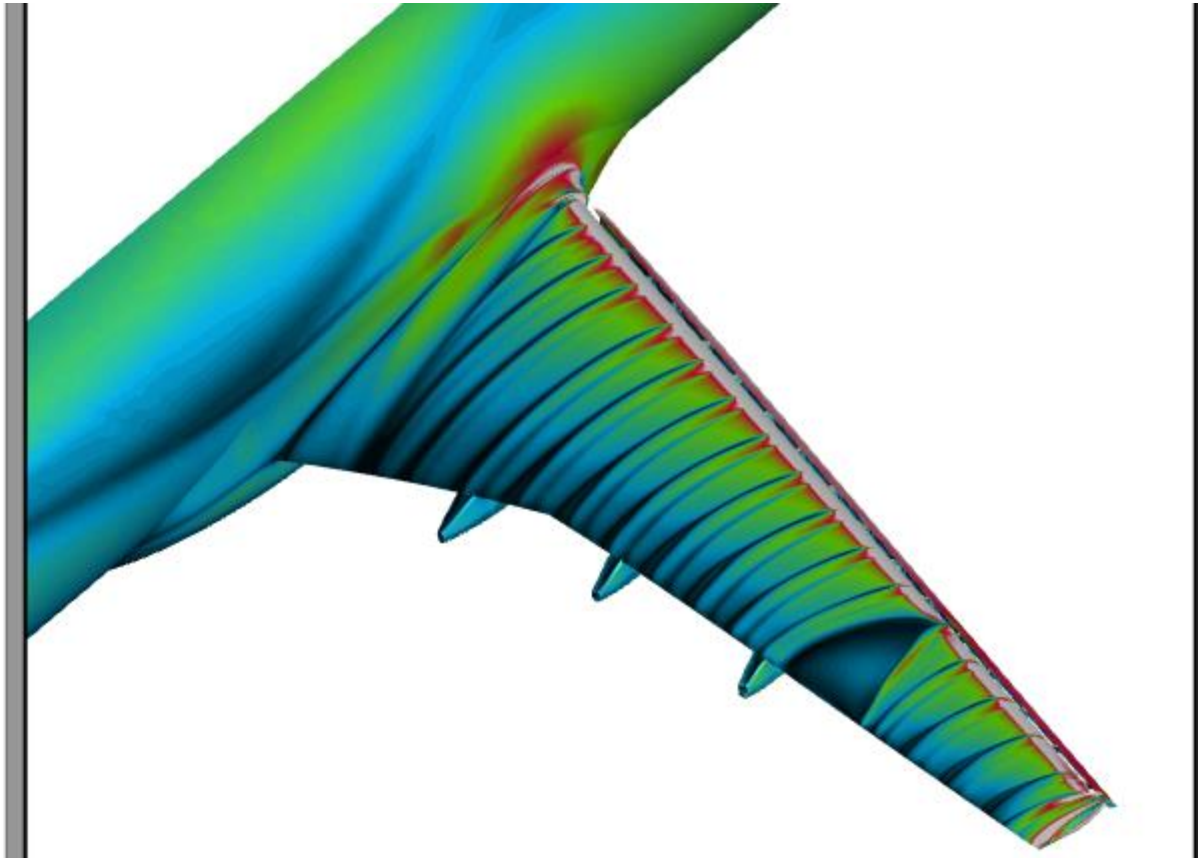


- Although not a priori obvious, running with different functional for error estimate has led to the same separation branch and $h \to 0$ limit
- Adapting to C_M improves convergence for this output at the cost of less aggressive resolution of the BL (cf. $C_{D,v}$ plot)

Fixed grid runs

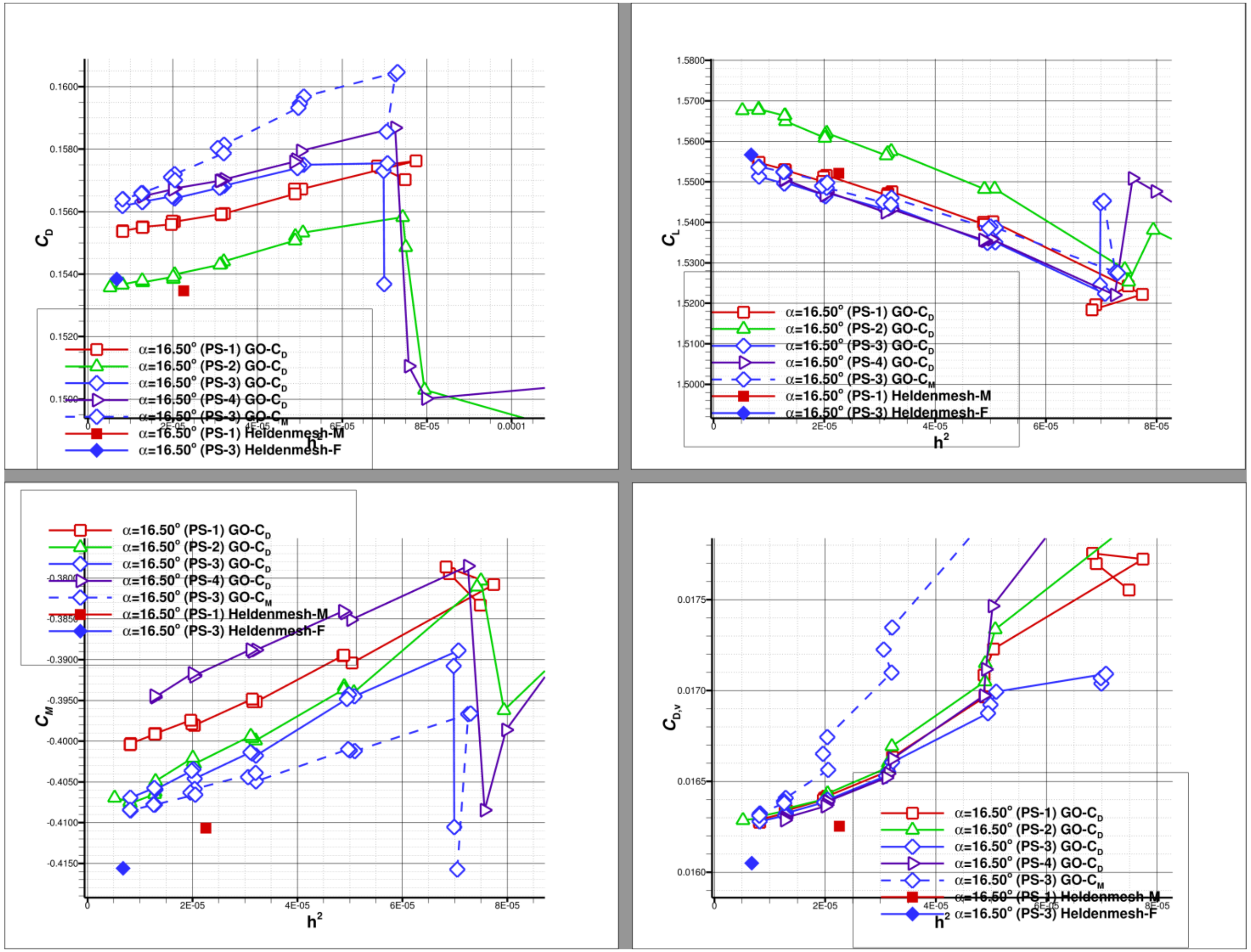


T1 Case 2.2, $\alpha = 16.5°$, Heldenmesh medium, SA-neg-noft2



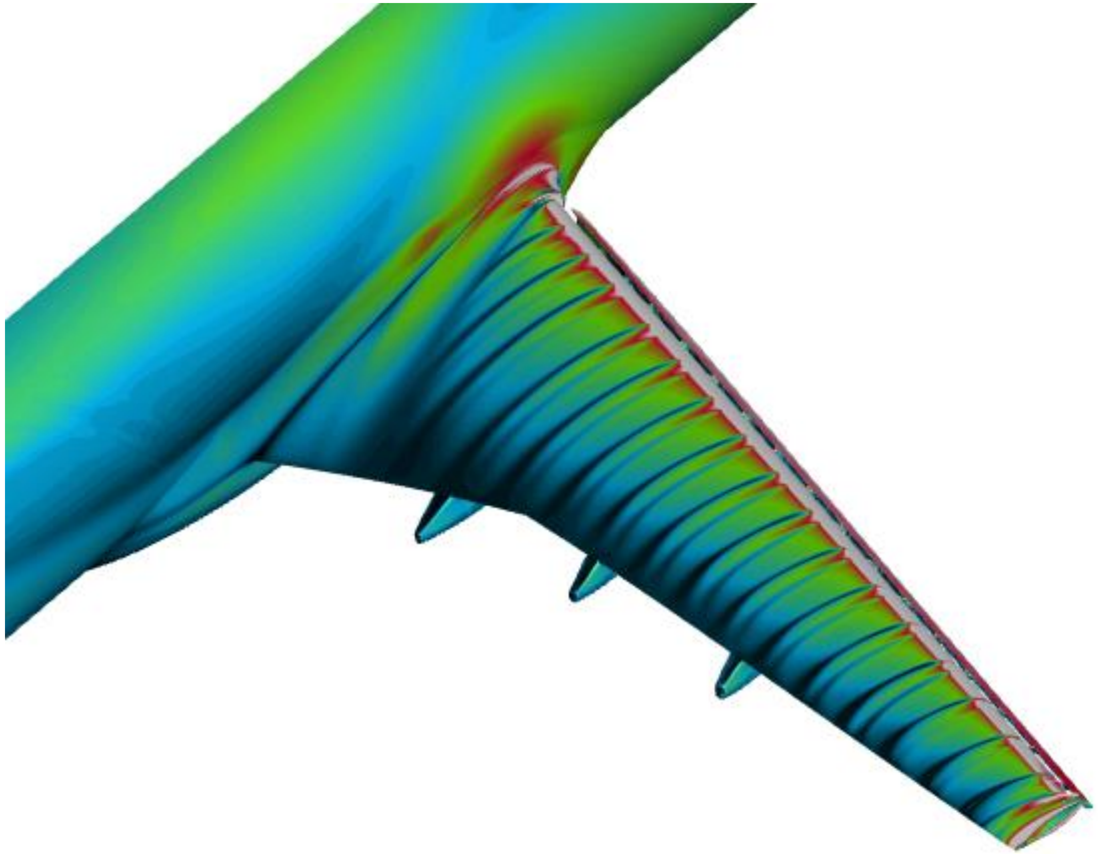
T1 Case 2.2, $\alpha = 16.5°$, Heldenmesh fine, SA-neg-noft2

Grid convergence with added fixed-grid datapoints

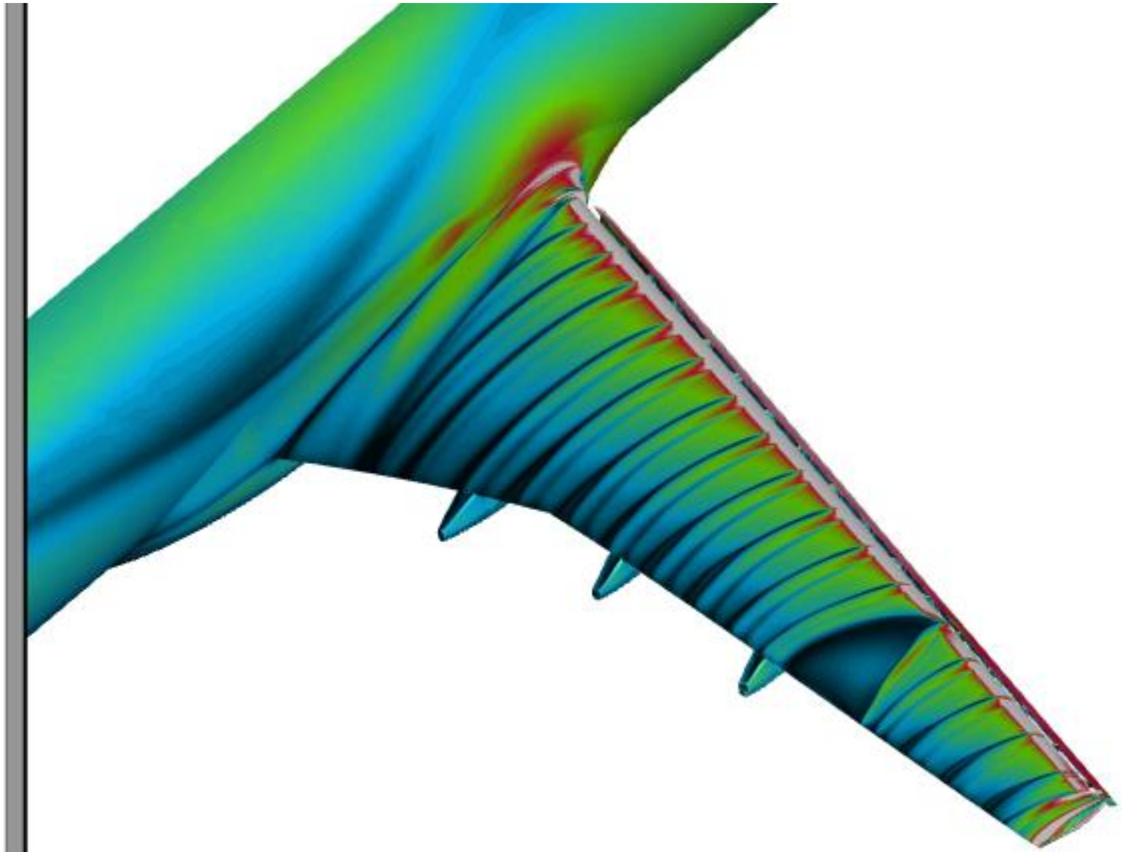


- All fixed grid were run from scratch (“cold start”)
- Fully residual-converged fixed-grid solutions qualitatively fell within a set of solutions obtained with adaptive approach
- Quantitatively, in terms of forces and moments, the agreement was not very good

Fixed grids: effects of residual convergence

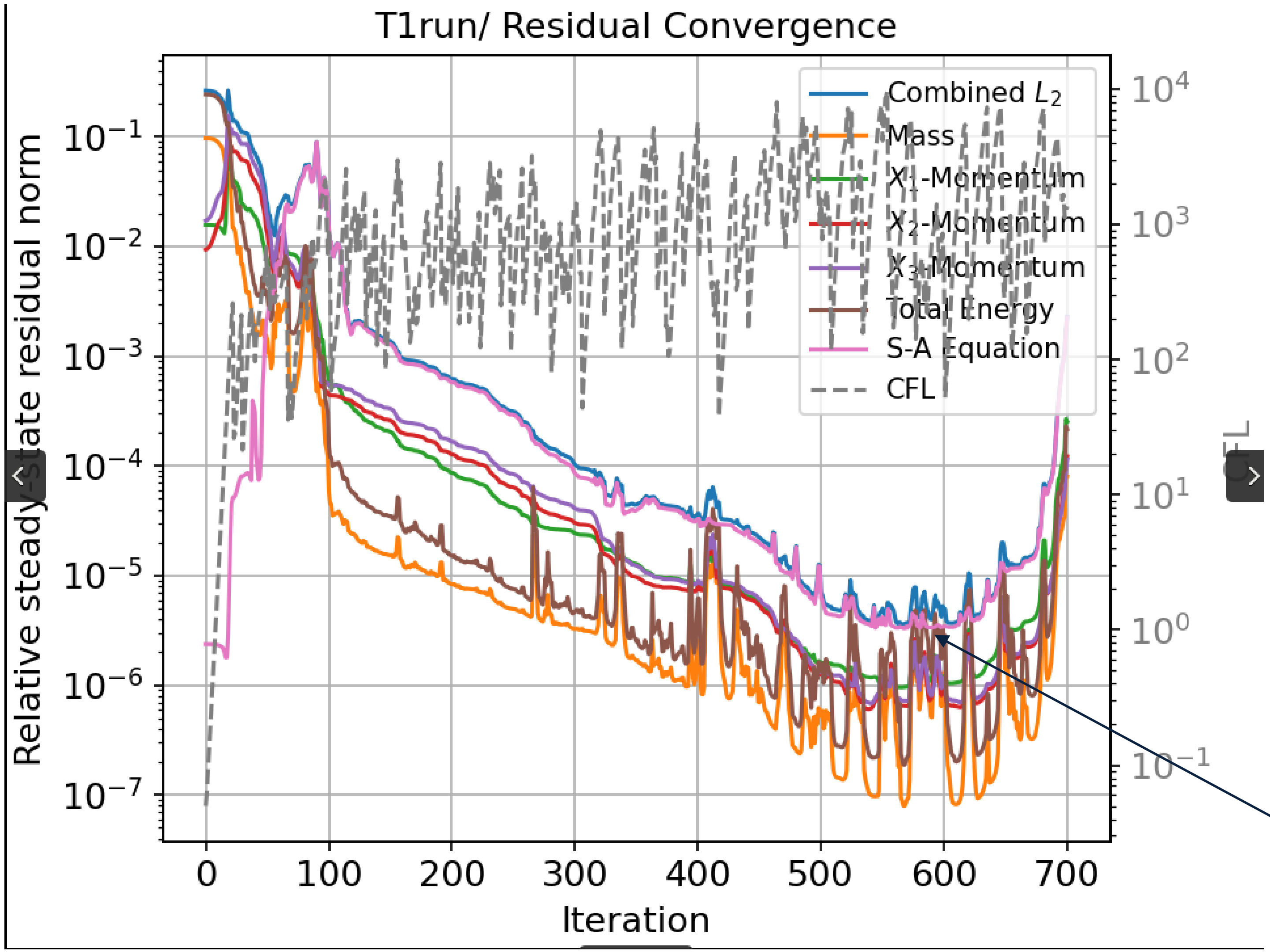


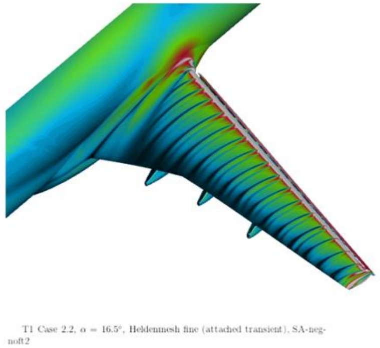
T1 Case 2.2, $\alpha = 16.5°$, Heldenmesh fine (attached transient), SA-neg-noft2



T1 Case 2.2, $\alpha = 16.5°$, Heldenmesh fine (converged), SA-neg-noft2

Attached-flow "pseudo-solution"





Incomplete. 5.5-orders iterative convergence

Questions/Conclusions/Discussion (I)

Best attempts to reduce numerical errors did not result in disappearance of Λ-solutions

- Does this mean that efforts to model and prevent PS phenomenon should be concentrated on the PDE/turbulence modelling level?

Is there a relation between "spurious separation" and the fact that we see these solutions in the amount of more than one?

It is desirable that RANS PDEs allow for multiple solutions to reflect real hysteresis effects near Cl-max

Questions/Conclusions/Discussion (II)

Does spurious Λ-separation violate any fundamental physical principle? Are Λ-separated solutions "unphysical" or just incorrectly representing real flows?

Are Λ-solutions "shadowing" attached (or properly, in-board-separated) solutions which (may?) exist but are "too weak attractors" for our solver strategies? Will getting rid of the former expose the latter?

- Assuming mathematical existence of Λ-separated solutions for PDE, can they be sorted out by carefully tailoring discrete solver strategies?

Multiple solutions individually behave like unique solutions of the well-posed problems: continuous behavior wrt. boundary conditions.

- Here we have demonstrated it for perturbations of α
- Although requires harder effort, this also can be demonstrated for small geometric perturbations